Tennessee Comprehensive Assessment Program / Mathematics

TCAP/CRA 2012-2013

4

Task 4: Adding and Multiplying Mixed Numbers Task

Full Scoring Guide

Task 4. Adding and Multiplying Mixed Numbers Task

$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.

REVIEW YOUR WORK IF YOU HAVE TIME.

4. Adding and Multiplying Mixed Numbers Task Scoring Guide

The CCSS for Mathematical Content (3 points)

4.NF.3b The student provides two ways to decompose the whole numbers and non-unit fractions into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$. _____

4.NF.3c The student accurately adds mixed numbers with like denominators. _____

4.NF.4a The student uses an equation to represent $\frac{a}{3}$ as $a \times \frac{1}{3}$ or any equivalent multiplication equation using $\frac{1}{3}$ as a factor, e.g, $8 \times \frac{1}{3} + 4 \times \frac{1}{3} = \frac{12}{3}$. _____

Total Content Points _____

The CCSS for Mathematical Practice (2 points)

MP1 The student writes two accurate addition equations and one accurate multiplication equation; completes all parts of the task. _____

(MP1: Make sense of problems and persevere in solving them.)

MP7 The student demonstrates that whole numbers and/or non-unit fractions can be decomposed and represented by equivalent fractions that comprise the whole or provides equivalent fractions that represent the whole numbers. _____

(MP7: Look for and make use of structure.)

Total Practice Points _____

Total Awarded Points _____

The CCSS for Mathematical Content Addressed in This Task

Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.

Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.

4.NF.3b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

4.NF.3c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

4.NF.4a Understand a fraction *a/b* as a multiple of *1/b*. *For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4).*

The CCSS for Mathematical Practice*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*Gray text indicates Mathematical Practices that are not addressed in this task.

Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

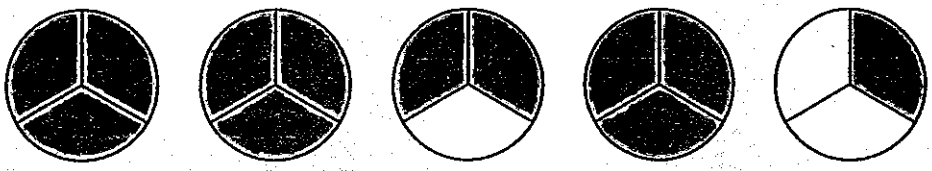
Task 4. Adding and Multiplying Mixed Numbers Task

$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2
6/3 + 4/3 = 12/3 = 4	15/3 + 4/3 = 19/3 = 4

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.



12 x 1/3 = 4

STOP

REVIEW YOUR WORK IF YOU HAVE TIME.

Litho#: 9639

Guide 1 Litho 9639

Total Content Points: 2 (4.NF.3c, 4.NF.4a)

Total Practice Points: 2 (MP1, MP7)

The student accurately adds mixed numbers with like denominators (4.NF.3c) and includes an accurate multiplication equation ($12 \times \frac{1}{3} = 4$) to represent $\frac{a}{3}$ as $a \times \frac{1}{3}$ using $\frac{1}{3}$ as a factor (4.NF.4a). The student writes two accurate addition equations and one accurate multiplication equation, completing all parts of the task (MP1).The student demonstrates that whole numbers and non-unit fractions can be decomposed and represented by equivalent fractions that comprise the whole (MP7). The student has rewritten the mixed numbers as improper fractions, rather than providing two ways to decompose the whole numbers and non-unit fractions into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ (no credit for 4.NF.3b).

Total Awarded Points: 4 out of 5

Task 4. Adding and Multiplying Mixed Numbers Task

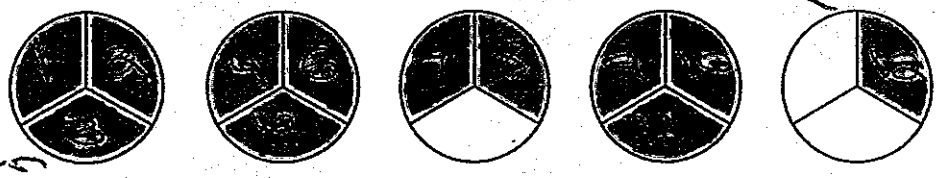
$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2
$\frac{8}{3} + \frac{4}{3} = \frac{12}{3} = 4$	$\frac{2}{3} + \frac{1}{3} + \frac{6}{3} + \frac{3}{3} = \frac{12}{3} = 4$

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.

4 shaded wholes altogether



$\frac{1}{3} \times \frac{12}{1} = 4$

STOP

REVIEW YOUR WORK IF YOU HAVE TIME.

Litho#: 9640

Guide 2 Litho 9640

Total Content Points: 2 (4.NF.3c, 4.NF.4a)

Total Practice Points: 2 (MP1, MP7)

The student does not successfully decompose the two mixed numbers given into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ two different ways, instead using the improper fractions $\frac{8}{3}$, $\frac{4}{3}$, and $\frac{6}{3}$ (no credit for 4.NF.3b). The student does correctly add mixed numbers with the same denominator (4.NF.3c) and use a multiplication equation to represent the fraction model in Part B (4.NF.4a). The response contains two correct equations finding the sum of 4 and a correct multiplication equation, so the student has responded appropriately to all parts of the task (MP1). The addition and multiplication shown indicate that the student understands that whole numbers and non-unit fractions can be decomposed and represented by equivalent fractions that comprise the whole (MP7).

Total Awarded Points: 4 out of 5

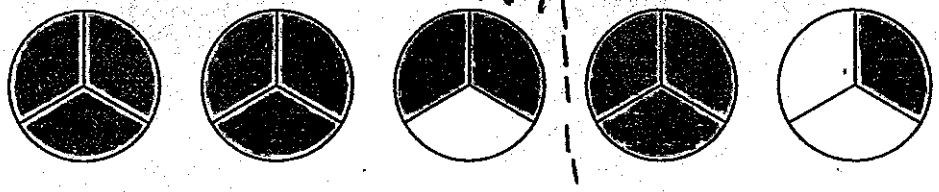
Task 4. Adding and Multiplying Mixed Numbers Task

$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2
13/3 − 1/3 = 12/3 = 4	1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3 = 8/3 + 1/3 + 1/3 + 1/3 + 1/3 = 12/3 = 4

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.



1/3 x 12 = 4

STOP

REVIEW YOUR WORK IF YOU HAVE TIME.

Litho#: 9648

Guide 3 Litho 9648

Total Content Points: 2 (4.NF.3c, 4.NF.4a)

Total Practice Points: 1 (MP7)

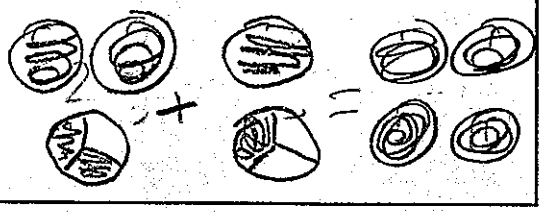
The student accurately adds and subtracts mixed numbers with like denominators (4.NF.3c). A multiplication equation ($\frac{1}{3} \times 12 = 4$) is used to represent $\frac{a}{3}$ as $a \times \frac{1}{3}$ with $\frac{1}{3}$ a factor (4.NF.4a). The student also demonstrates that whole numbers and non-unit fractions can be decomposed and represented by equivalent fractions that comprise the whole (MP7). The student does not provide two clearly defined ways to decompose the whole numbers and non-unit fractions into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ (no credit for 4.NF.3b). The student writes two equations using the unit of thirds, which both equal 4, but the first equation does not reflect the problem from the task, indicating that the student has not fully made sense of and completed the task (no credit for MP1).

Total Awarded Points: 3 out of 5

Task 4. Adding and Multiplying Mixed Numbers Task

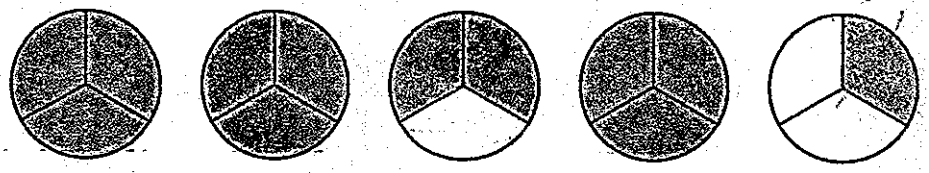
$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2
$2\frac{2}{3} + 1\frac{1}{3} = 4$	

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.

4 wholes



$12 \times \frac{1}{3} = 4$

STOP

REVIEW YOUR WORK IF YOU HAVE TIME.

Litho#: 9525

Guide 4 Litho 9525

Total Content Points: 2 (4.NF.3c, 4.NF.4a)

Total Practice Points: 0

The student accurately adds mixed numbers with like denominators (4.NF.3c) and provides a multiplication equation ($12 \times \frac{1}{3} = 4$) to represent $\frac{a}{3}$ as $a \times \frac{1}{3}$ using $\frac{1}{3}$ as a factor (4.NF.4a). The response does not include two accurate addition equations, and thus, does not complete all parts of the task (no credit for MP1). The response does not demonstrate that whole numbers and/or non-unit fractions can be decomposed and represented by equivalent fractions that comprise the whole (no credit for MP7). Both equations show the original whole numbers and non-unit fractions and not two different ways to group them into thirds (no credit for 4.NF.3b).

Total Awarded Points: 2 out of 5

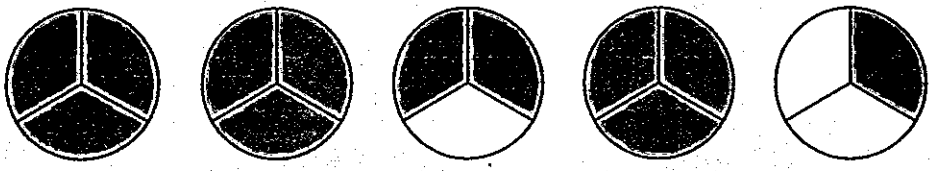
Task 4. Adding and Multiplying Mixed Numbers Task

$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2
8/3 4/3 = 12/3 = 4	2+1=3+1=4 1/3+2/3=1

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.



1/3 x 3 = 3/3 = 1 + 2/3 = 1 2/3

REVIEW YOUR WORK IF YOU HAVE TIME.

Litho#: 9637

Guide 5 Litho 9637

Total Content Points: 1 (4.NF.3c)

Total Practice Points: 1 (MP7)

The student accurately adds mixed numbers with like denominators (4.NF.3c) and demonstrates that whole numbers and non-unit fractions can be decomposed and represented by equivalent fractions that comprise the whole (MP7). The student only provides one way to decompose the whole numbers and non-unit fractions into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ (no credit for 4.NF.3b), and does not write an equation to represent $\frac{a}{3}$ as $a \times \frac{1}{3}$ or any equivalent multiplication equation using $\frac{1}{3}$ as a factor (no credit for 4.NF.4a). The student has not written two appropriate addition equations, as the second equation uses whole numbers rather than only the unit of thirds (no credit for MP1).

Total Awarded Points: 2 out of 5

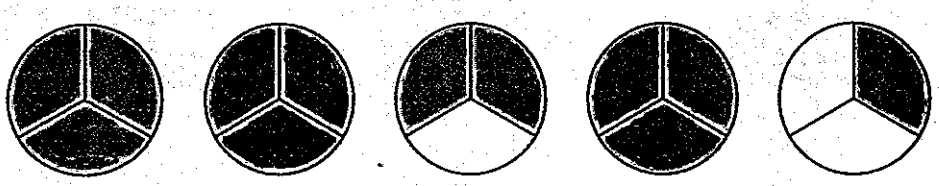
Task 4. Adding and Multiplying Mixed Numbers Task

$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2
$\frac{8}{3} + \frac{4}{3} = \frac{12}{3}$	$\left(\frac{2}{3} + \frac{1}{3}\right) + \left(\frac{6}{3} + \frac{3}{3}\right) = \frac{12}{3}$

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.



$\frac{3}{3} \times 4 = 4$

STOP

REVIEW YOUR WORK IF YOU HAVE TIME.

Litho#: 9644

Guide 6 Litho 9644

Total Content Points: 1 (4.NF.3c)

Total Practice Points: 1 (MP7)

The student accurately adds mixed numbers with like denominators (4.NF.3c), but provides a multiplication equation ($\frac{3}{3} \times 4 = 4$) that does not use $\frac{1}{3}$ as a factor, and thus does not clearly show that $\frac{a}{3}$ can be represented as $a \times \frac{1}{3}$ (no credit for 4.NF.4a). The response does not include an appropriate multiplication equation using $\frac{1}{3}$ as a factor and thus, the student does not complete all parts of the task (no credit for MP1). The addition equations in Part A does demonstrate that whole numbers and/or non-unit fractions can be decomposed and represented by equivalent fractions that comprise the whole, as the mixed numbers from the expression in the task are represented by different fractions with 3 as the denominator (MP7). Not provided are two ways to decompose the whole numbers and non-unit fractions into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ (no credit for 4.NF.3b).

Total Awarded Points: 2 out of 5

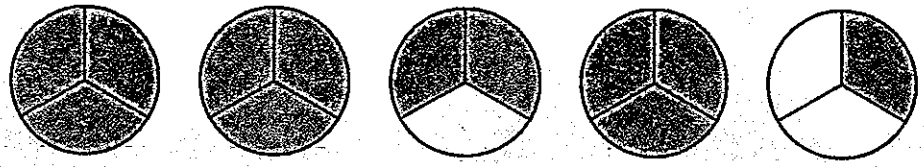
Task 4. Adding and Multiplying Mixed Numbers Task

$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2
$2\frac{2}{3} + 1\frac{1}{3} = 3\frac{3}{3}$	

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.



REVIEW YOUR WORK IF YOU HAVE TIME.

Litho#: 9498

Guide 7 Litho 9498

Total Content Points: 1 (4.NF.3c)

Total Practice Points: 0

The student accurately adds mixed numbers with like denominators (4.NF.3c). The student does not provide two ways to decompose the whole numbers and non-unit fractions into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ (no credit for 4.NF.3b), nor use an equation to represent $\frac{a}{3}$ as $a \times \frac{1}{3}$ or any equivalent multiplication equation with $\frac{1}{3}$ as a factor (no credit for 4.NF.4a). The student does not write two accurate addition equations and one accurate multiplication equation to complete all parts of the task (no credit MP1). No work is shown to demonstrate that whole numbers and/or non-unit fractions can be decomposed and represented by equivalent fractions that comprise the whole, or that equivalent fractions can be used to represent whole numbers (no credit MP7).

Total Awarded Points: 1 out of 5

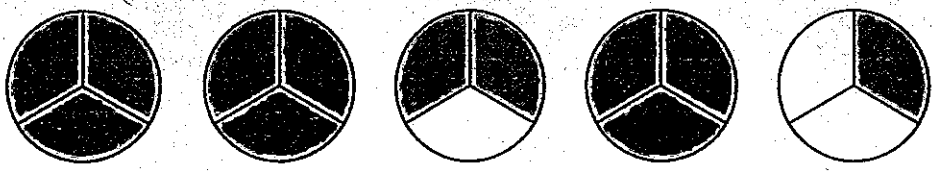
Task 4. Adding and Multiplying Mixed Numbers Task

$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2
2/3 + 1/3 + 1/3 + 1/3 + 2/3 + 1/3 + 1/3 = 3	1/3 + 1/3 + 1/3 + 2/3 + 1/3 + 1/3 + 1/3 = 3

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.



2 + 2/3 + 1 + 1/3 = 4

1

STOP

REVIEW YOUR WORK IF YOU HAVE TIME.

Litho#: 9642

Guide 8 Litho 9642

Total Content Points: 1 (4.NF.3c)

Total Practice Points: 0

The student correctly adds mixed numbers in Part B (4.NF.3c). The student does not decompose the fractions from the given expression into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ to show two different ways to find a solution (no credit for 4.NF.3b). There is no multiplication equation to represent the model shown in Part B (no credit for 4.NF.4a). The student has not appropriately responded to all parts of the task, as there is no multiplication equation and the addition does not match the expression given (no credit for MP1). Separating whole numbers from fractions in the mixed numbers in Part B is not enough to demonstrate that whole numbers and non-unit fractions can be decomposed and represented by equivalent fractions, and as the student has not decomposed the mixed numbers given in Part A, a full understanding of the structure of fractions has not been demonstrated (no credit for MP7).

Total Awarded Points: 1 out of 5

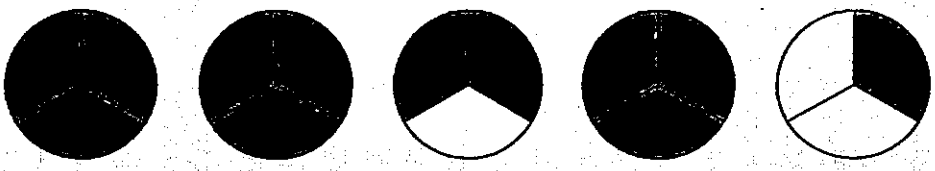
Task 4. Adding and Multiplying Mixed Numbers Task

$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2
$2\frac{2}{3} + 1\frac{1}{3} = 4$	$2\frac{2}{3} - 1\frac{1}{3} = 1\frac{1}{3}$

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.



$\frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3} + \frac{3}{3} + \frac{1}{3} = 8$ or $\frac{15}{3}$

REVIEW YOUR WORK IF YOU HAVE TIME.

Guide 9 Litho 13137

Total Content Points: 1 (4.NF.3c)

Total Practice Points: 0

The student accurately adds and subtracts mixed numbers with like denominators in parts A and B (4.NF.3c). The student does not provide two ways to decompose the whole numbers and non-unit fractions into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ (no credit for 4.NF.3b), nor write an equation to represent $\frac{a}{3}$ as $a \times \frac{1}{3}$ or any equivalent multiplication equation using $\frac{1}{3}$ as a factor (no credit for 4.NF.4a). The student does not write two accurate addition equations and one accurate multiplication equation; does not complete all parts of the task (no credit for MP1) and does not demonstrate that whole numbers and/or non-unit fractions can be decomposed and represented by equivalent fractions that comprise the whole or provide equivalent fractions that represent the whole numbers (no credit for MP7).

Total Awarded Points: 1 out of 5

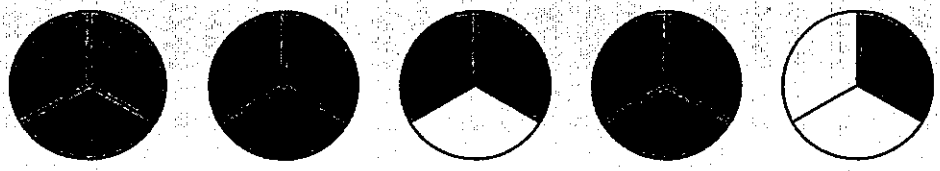
Task 4. Adding and Multiplying Mixed Numbers Task

$$2\frac{2}{3} + 1\frac{1}{3} =$$

a. Partition the whole numbers and fractions into thirds in different ways. Write 2 different equations based on the problem above using only the unit of thirds.

Equation 1	Equation 2

b. Look at the picture below. Write a multiplication equation that describes the picture and uses $\frac{1}{3}$ as a factor.



$12 \times \frac{1}{3}$

STOP

REVIEW YOUR WORK IF YOU HAVE TIME.

Guide 10 Litho 13140

Total Content Points: 0

Total Practice Points: 0

The student has not provided two ways to decompose the whole numbers and non-unit fractions into groups of $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ (no credit for 4.NF.3b). No addition of mixed numbers with like denominators has been provided (no credit for 4.NF.3c). The student has not written an equation to represent $\frac{a}{3}$ as $a \times \frac{1}{3}$ or any equivalent multiplication equation using $\frac{1}{3}$ as a factor (no credit for 4.NF.4a). The student does include the expression $12 \times \frac{1}{3}$, but does not write accurate equations to complete the task (no credit for MP1). The student does not demonstrate that whole numbers and/or unit fractions can be decomposed and represented by equivalent fractions that comprise the whole or provide equivalent fractions that represent the whole numbers (no credit for MP7).

Total Awarded Points: 0 out of 0