Tennessee Comprehensive Assessment Program / Mathematics

TCAP/CRA PILOT 2012



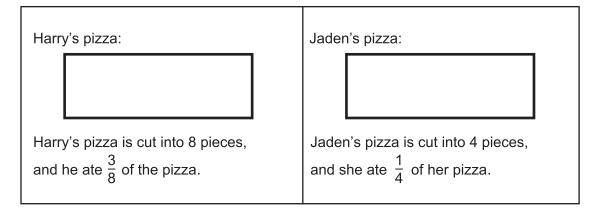
Task 1 : Pizza Scoring Guide

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Task 1. Pizza Task

Both Harry and Jaden ate a portion of the same size pizza.

a. Show the portion of pizza that each student ate.



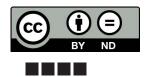
b. Use words and fractions to explain who ate the most pizza.

Page 6	GO ON TO THE NEXT PAGE.
 BY ND	

C.	Use fractions to explain how much more pizza one person ate than the
	other person.



Page 7



GO ON TO THE NEXT PAGE.

1. The Pizza Task Scoring Guide

The CCSS for Mathematical Content (1 point)

- 4.NF.2 Student identifies Harry as the person who ate the most pizza. Student may explain reasoning by:
 - explaining that $\frac{3}{8}$ is comprised of $\frac{2}{8} + \frac{1}{8}$; since $\frac{2}{8} = \frac{1}{4}$, this part of Harry's

pizza is equivalent to Jaden's. Harry, therefore, has $\frac{1}{8}$ more pizza than Jaden.

- shading on the students' diagram of the pizza is accurate and indicates that $\frac{2}{3}$

of the $\frac{3}{8}$ of Harry's pizza is equal to $\frac{1}{4}$ of Jaden's pizza because they have the same shaded portion/area of the pizza.

Total Content Points _____

The CCSS for Mathematical Practices (4 points)

MP1 The student represents each amount with a diagram, and indicates that Harry ate the most pizza and specifies how much more was eaten by Harry than by Jaden.(MP1: Make sense of problems and persevere in solving them.)

MP4 The model shows an understanding of the relationship between fourths and eighths.

- Student diagrams show that fourths are drawn as double the size/portion of eighths, thus $\frac{1}{4}$ is equivalent to $\frac{2}{8}$.
- The fraction notation indicates a comparison of the amounts. $\frac{3}{8} > \frac{1}{4}$ or $\frac{1}{4} < \frac{3}{8}$.

(MP4: Model with mathematics.)

- MP6 Accurate and precise work is shown.
 - Drawing shows correct number of equal size pieces.
 - Student identifies $\frac{3}{8}$ as more pizza than $\frac{1}{4}$; may write $\frac{3}{8} > \frac{1}{4}$.
 - Student indicates that Harry ate $\frac{1}{8}$ more pizza than Jaden.

(MP6: Attend to precision.)

- MP7 Work indicates that the student recognizes the two fractions can only be compared if they have like denominators or if the same portion/area of the figure is shaded. Work may show:
 - $\frac{2}{8}$ is equal to $\frac{1}{4}(\frac{2}{8} = \frac{1}{4})$
 - $\frac{2}{8}$ shaded on one diagram and $\frac{1}{4}$ shaded on the other and a claim that they are the same amount of pizza.
 - reason why pieces of like size need to be created in order to compare the amounts.

(MP7: Look for and make use of structure.)

Total Practice Points _____

Total Awarded Points _____

The CCSS for Mathematical Content Addressed in This Task

Extend understanding of fraction equivalence and ordering.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

The CCSS for Mathematical Practices*

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

* Gray text indicates Mathematical Practices not addressed in this task.

Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

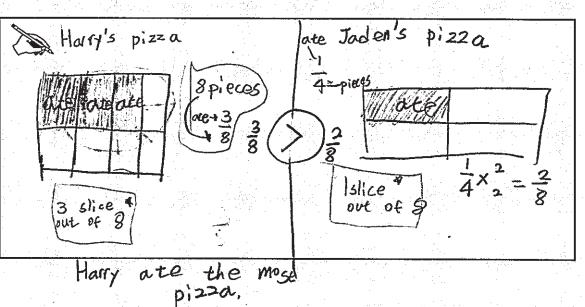
Task 1. Pizza Task

Both Harry and Jaden ate a portion of the same size pizza.

a. Show the portion of pizza that each student ate.

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and he ate	$\frac{3}{8}$ of the pix	zza.	and sh	the ate $\frac{1}{4}$ of the second seco	ner pizza	•	

b. Use words and fractions to explain who ate the most pizza.



Page 6GO ON TO THE NEXT PAGE.Litho#: 40357Page 6

Guide 1a

Guide 1b Refer to the pizzas and explain how much more pizza one person ate than the C. other person. 8 ate nt a Horry's Pizza Jadon's Pizza & pizza more, Harry ate Page 7 GO ON TO THE NEXT PAGE. 40357 Page 7 Litho#: 40357

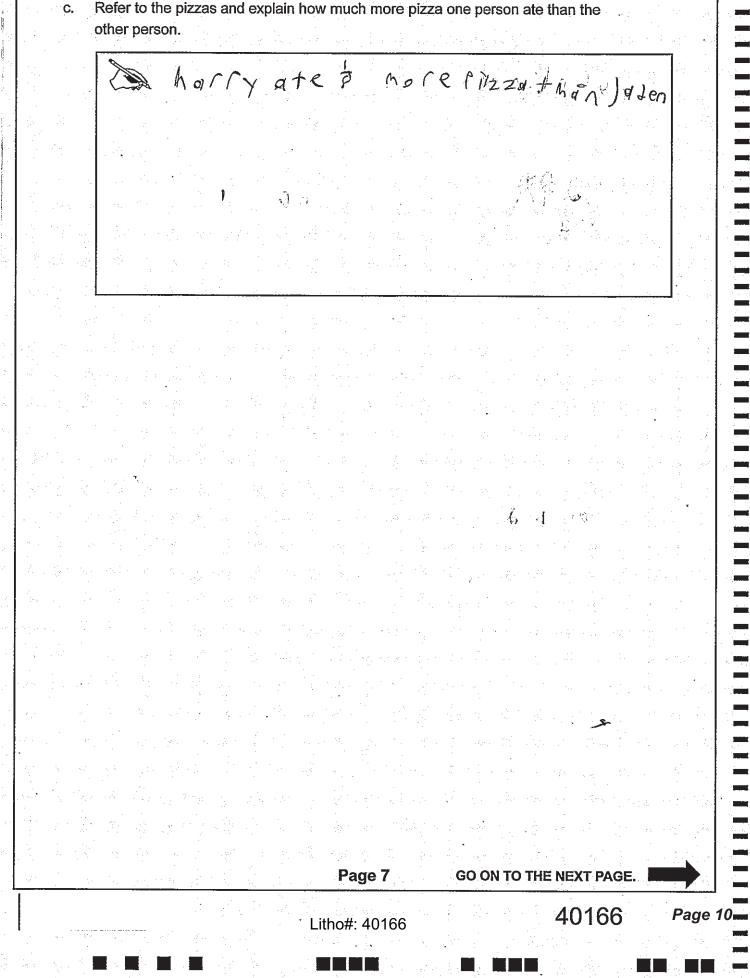
Guide 1	Litho. 40357
Total Content Points: 1	(4.NF.2)
Total Practice Points: 4	(MP1, MP4, MP6, MP7)

This response indicates that Harry is the person who ate the most pizza. The student's explanation shows equivalency with diagrams and by using words and fractions (4.NF.2). The student represents each amount with a diagram and indicates that Harry ate $\frac{1}{8}$ more pizza than Jaden (MP1). The given diagrams show that fourths are double the size/portion of eighths and the student additionally indicates that $\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$, then makes a fraction notation $\frac{3}{8} > \frac{2}{8}$ (MP4). The diagrams are accurate. The student identifies $\frac{3}{8}$ as more pizza than $\frac{1}{4}$ and indicates that Harry ate $\frac{1}{8}$ more of his pizza than Jaden ate of his (MP6). The student's work indicates a recognition that the two fractions need to be compared using like denominators (MP7).

Total Awarded Points: 5 out of 5

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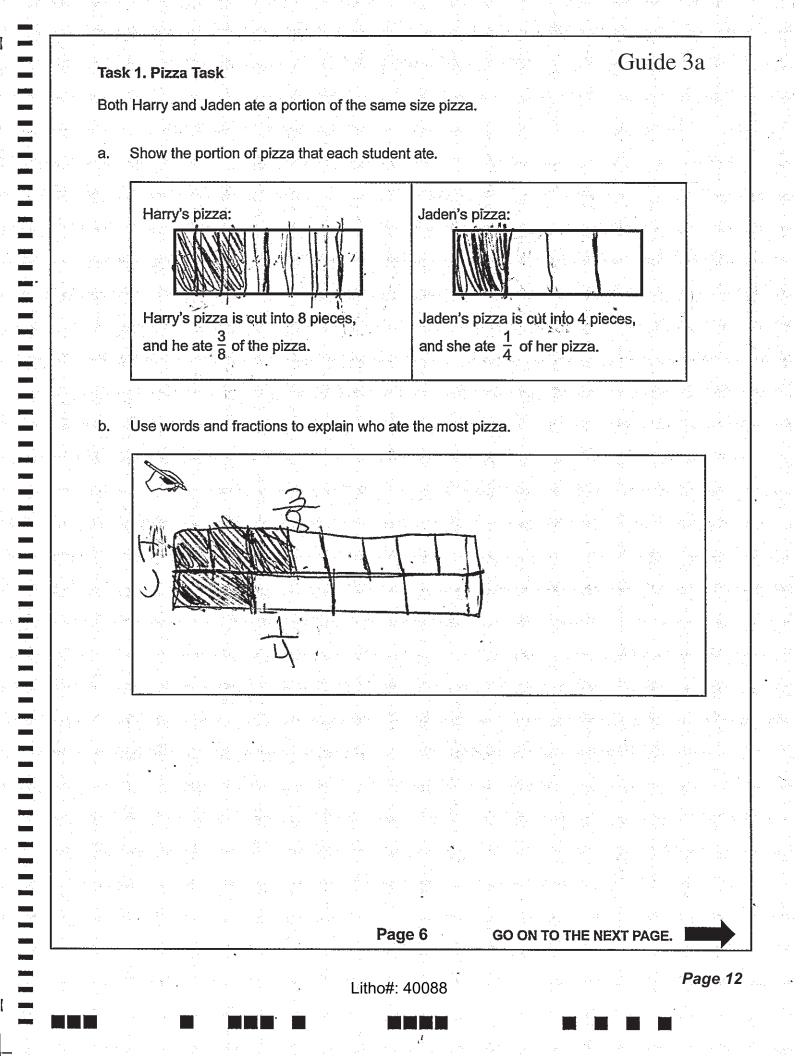
Guide 2b Refer to the pizzas and explain how much more pizza one person ate than the other person.

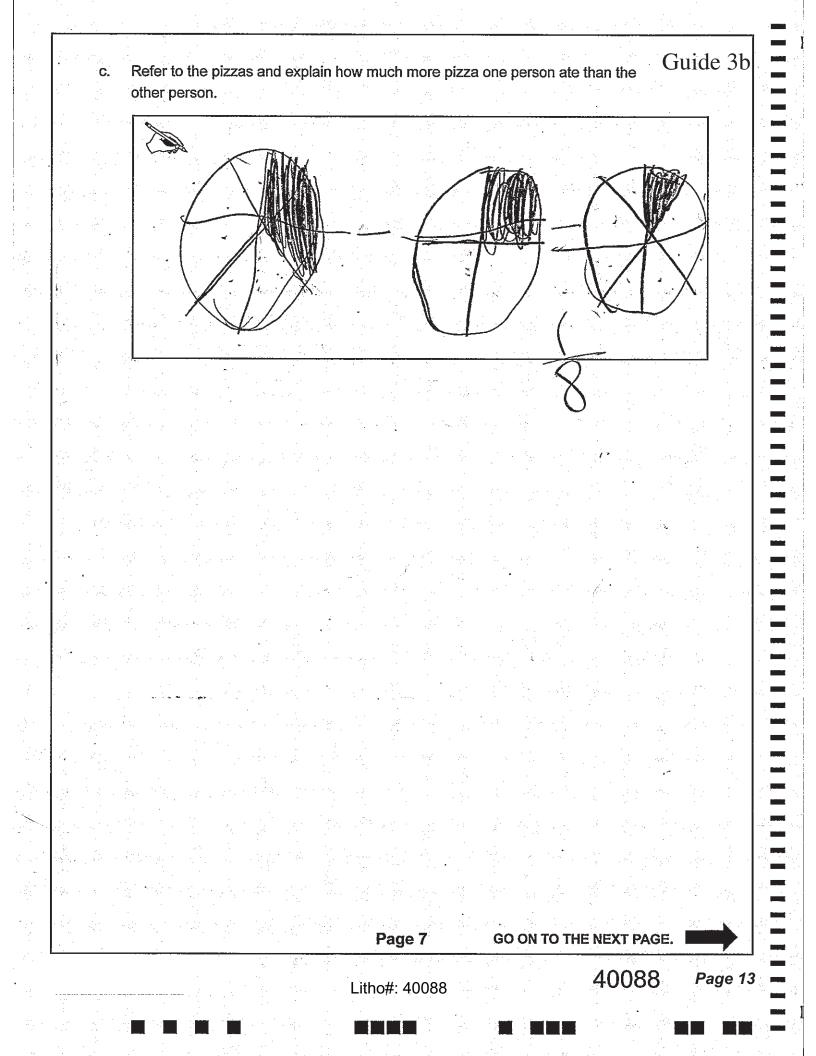


Guide 2Litho. 40166Total Content Points: 1(4.NF.2)Total Practice Points: 4(MP1, MP4, MP6, MP7)

This response indicates that Harry is the person who ate the most pizza. The student's explanation shows equivalency with diagrams, and by using words and fractions ("if Jaden's pizza had the same denomenator it would be $\frac{2}{8}$ insted of $\frac{1}{4}$. harry had $\frac{3}{8}$ and Jaden had $\frac{2}{8}$ ") (4.NF.2). The student represents each amount with a diagram and indicates that Harry ate $\frac{1}{8}$ more pizza than Jaden (MP1). The given diagrams show that fourths are double the size/portion of eighths (MP4). The diagrams are accurate and clearly identify that $\frac{3}{8}$ is greater than $\frac{1}{4}$. The student indicates that Harry ate $\frac{1}{8}$ more pizza than Jaden (MP6). The student's work indicates a recognition that the two fractions need to be compared using like denominators (MP7).

Total Awarded Points: 5 out of 5





Guide 3	Litho. 40088
Total Content Points: 1	(4.NF.2)
Total Practice Points: 3	(MP1, MP4, MP6)

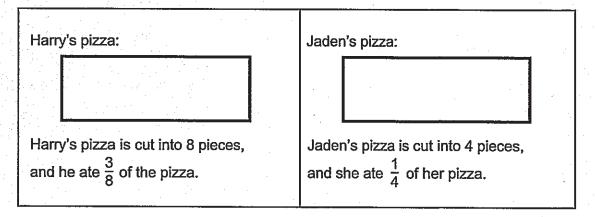
The student indicates that Harry is the person who ate the most pizza by labeling the accurately drawn diagram in part b as "H." The student indicates this visually by presenting accurate diagrams that show that $\frac{2}{8}$ of the $\frac{3}{8}$ of Harry's pizza is equal to $\frac{1}{4}$ of Jaden's (4.NF.2). The student represents each amount with a diagram, indicates that "H" ate more, and shows that $\frac{1}{8}$ of the pizza remains after subtraction (MP1). The student's diagrams show that fourths are double the size/portion of eighths (MP4). The student's diagrams are accurate and clearly identify that $\frac{3}{8}$ is greater than $\frac{1}{4}$ although the student does not definitively name Harry as the one who consumed the greater amount of pizza (MP6). Although the diagram in part b shows the two fractions should have the same denominator in order to be accurately compared, or that the amount shown by $\frac{1}{4}$ needs to equal the same amount as $\frac{2}{8}$ (no credit for MP7).

Total Awarded Points: 4 out of 5

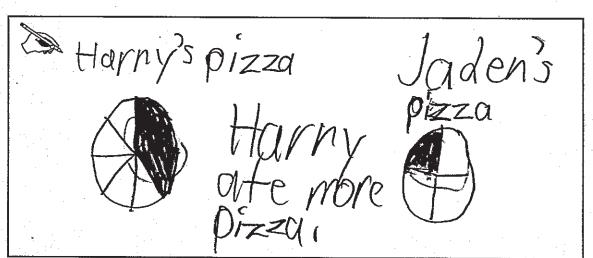
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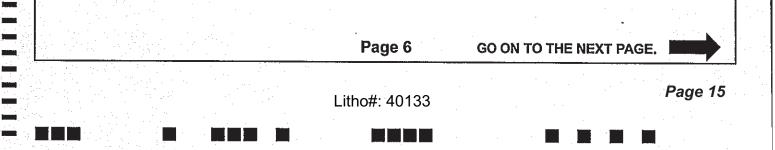
Both Harry and Jaden ate a portion of the same size pizza.

a. Show the portion of pizza that each student ate.

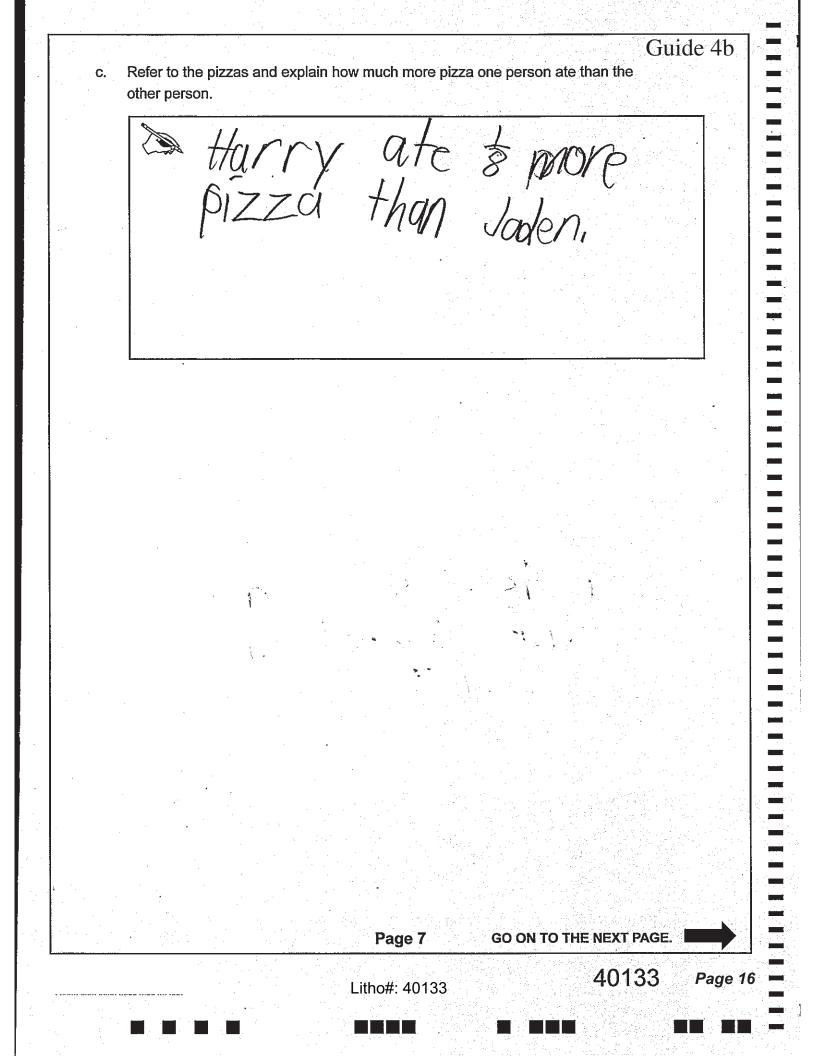


b. Use words and fractions to explain who ate the most pizza.





Guide 4a



Guide 4 Litho. 40133

Total Content Points: 1 (4.NF.2)

Total Practice Points: 3 (MP1, MP4, MP6)

The student indicates that Harry is the person who ate the most pizza and indicates this visually by presenting accurate diagrams showing that $\frac{2}{8}$ of Harry's pizza is equal to $\frac{1}{4}$ of Jaden's (4.NF.2). The student represents each amount with a diagram and indicates that Harry ate $\frac{1}{8}$ more pizza than Jaden (MP1). Although the diagrams are not written in part a, they are clear and correct, and show that fourths are double the size/portion of eighths (MP4). The diagrams are accurate and clearly identify that $\frac{3}{8}$ is greater than $\frac{1}{4}$. The student indicates that Harry ate $\frac{1}{8}$ more pizza than Jaden (MP6). Part b shows a visual model of the two pizzas, but the student has not explained that $\frac{1}{4}$ is equal to $\frac{2}{8}$, and that therefore the $\frac{1}{4}$ -pizza slice must be the same size as $2\frac{1}{8}$ -pizza slices. Without that explanation, or a clear comparison of the two fractions indicating that fractions can be subtracted only with like denominators, the student has not clearly demonstrated an understanding of the structure of fractions that would allow him to solve other problems involving fractional amounts of a whole (no credit for MP7).

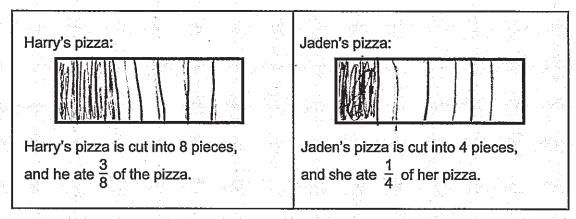
Total Awarded Points: 4 out of 5

Guide 5a

Task 1. Pizza Task

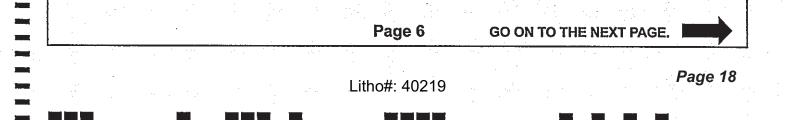
Both Harry and Jaden ate a portion of the same size pizza.

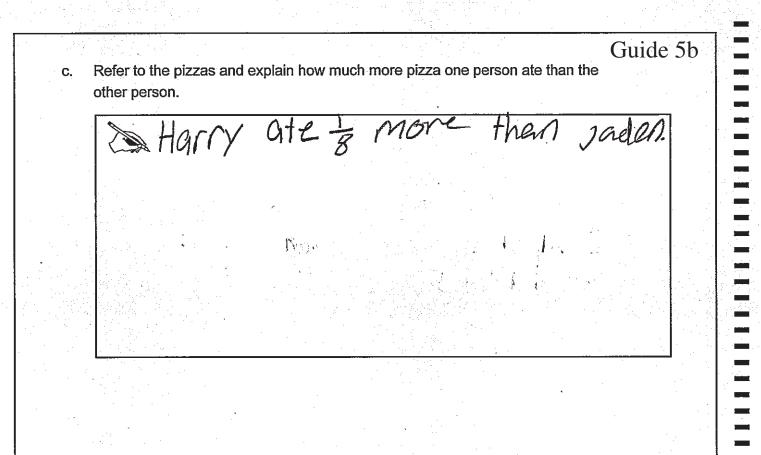
a. Show the portion of pizza that each student ate.

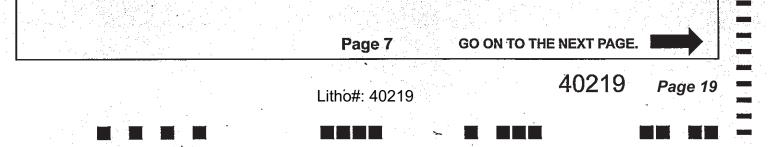


b. Use words and fractions to explain who ate the most pizza.

Hampsin Atc, BPSC because 15 bigger then it.







Guide 5

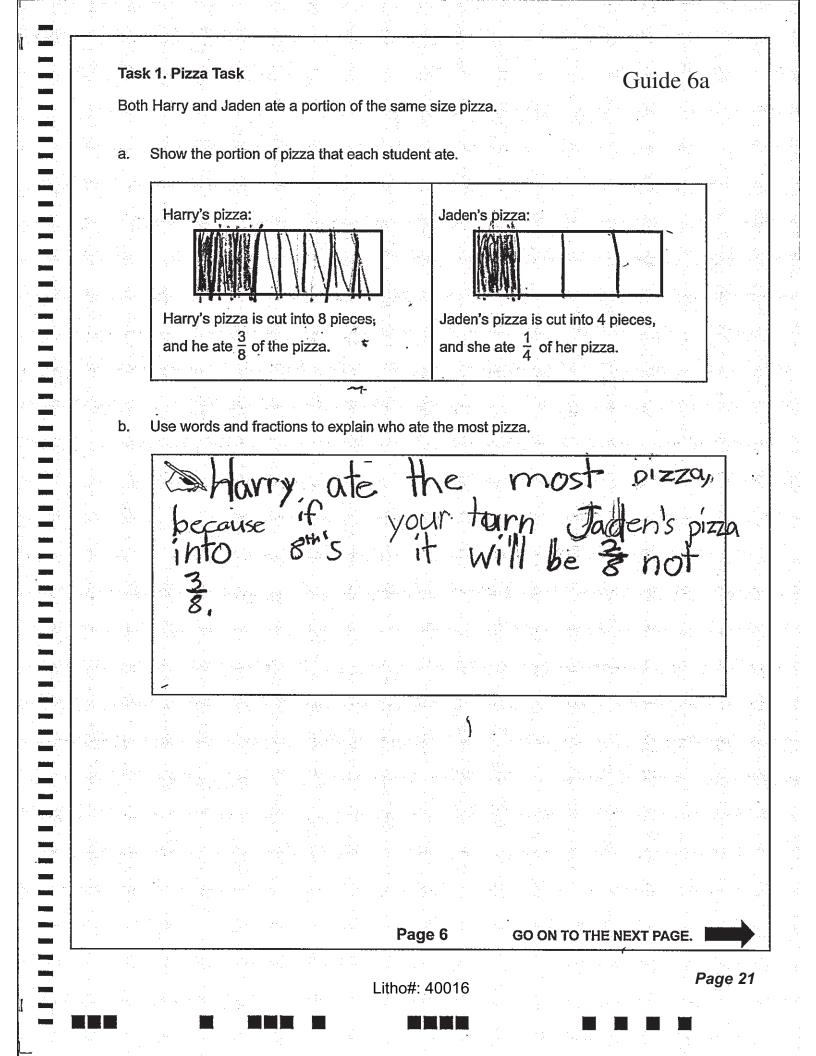
Litho. 40219

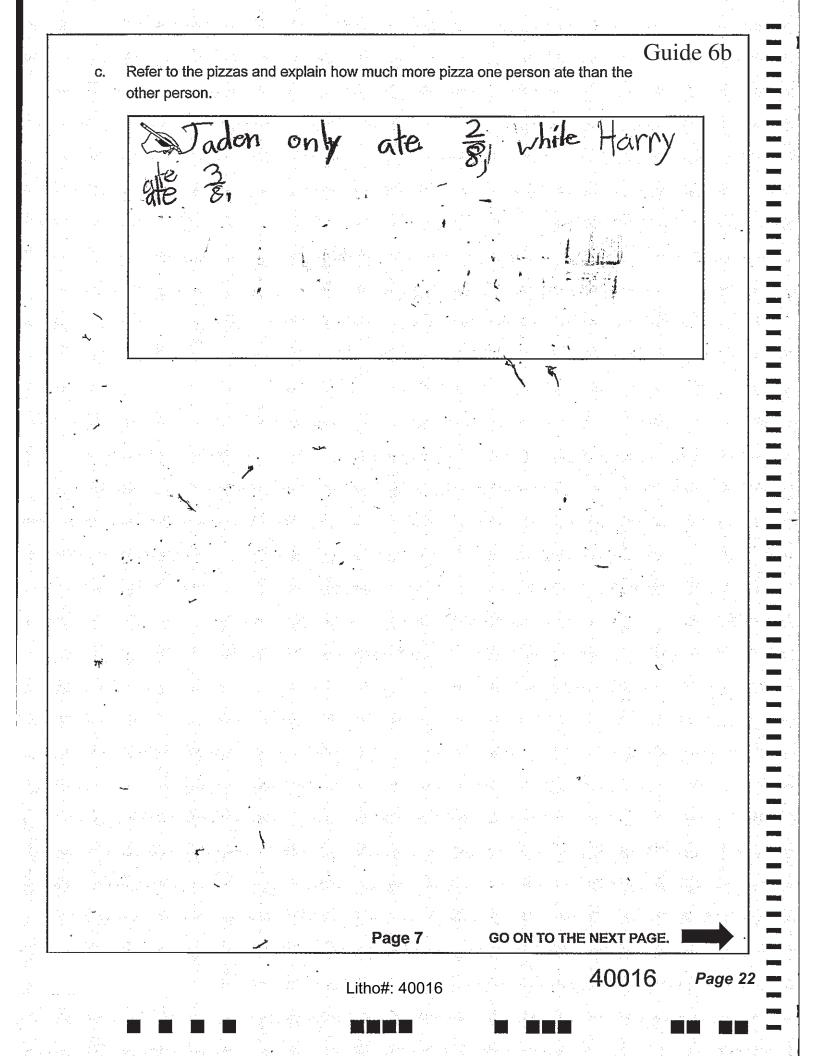
Total Content Points: 1 (4.NF.2)

Total Practice Points: 3 (MP1, MP4, MP6)

The student indicates that Harry is the person who ate the most pizza and indicates this visually by presenting accurate diagrams showing that $\frac{2}{8}$ of Harry's pizza is equal to $\frac{1}{4}$ of Jaden's (4.NF.2). The student represents each amount with a diagram and indicates that Harry ate $\frac{1}{8}$ more pizza than Jaden (MP1). The diagrams show that fourths are double the size/portion of eighths (MP4). The student has accurate drawings, identifies $\frac{3}{8}$ as more pizza than $\frac{1}{4}$, and indicates that Harry ate $\frac{1}{8}$ more (MP6). The student has drawn a roughly accurate model of the two pizzas in part a, and has given the correct difference in part c, but she has not provided an explanation that recognizes that the size of a $\frac{1}{4}$ -pizza piece must be the same size as two $\frac{1}{8}$ -pizza pieces (no credit for MP7).

Total Awarded Points: 4 out of 5





Guide 6	Litho. 40016
Total Content Points: 1	(4.NF.2)
Total Practice Points: 2	(MP4, MP7)

The student indicates that Harry is the person who ate the most pizza and shows his reasoning visually by presenting accurate diagrams showing that $\frac{2}{8}$ of Harry's pizza is equal to $\frac{1}{4}$ of Jaden's (4.NF.2). The student represents each amount with a diagram and indicates that Harry ate the most pizza, but does not specifically indicate how much more Harry ate (no credit for MP1). The student's diagrams show that fourths are double the size/portion of eighths (MP4). The student has accurate drawings and identifies $\frac{3}{8}$ as more pizza than $\frac{1}{4}$ ("turn Jaden's pizza into 8th's it will be $\frac{2}{8}$...Jaden only ate $\frac{2}{8}$, while Harry ate $\frac{3}{8}$ ") but does not specify how much more pizza Harry ate than Jaden, and does not show any equations, which limits the level of precision in the response (no credit for MP6). Work indicates that the student recognizes the two fractions need to be compared using like denominators ("if your turn Jaden's pizza into 8th's it will be $\frac{2}{8}$ not $\frac{3}{8}$ ") (MP7).

Total Awarded Points: 3 out of 5

Task 1. Pizza Task

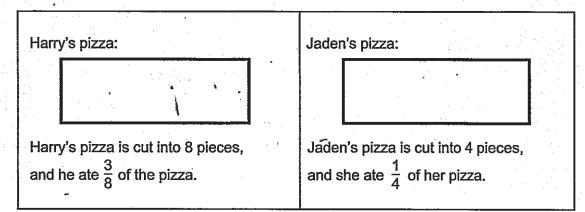
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b.

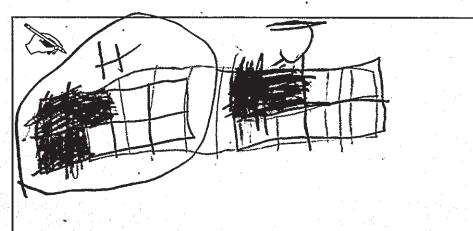
Guide 7a

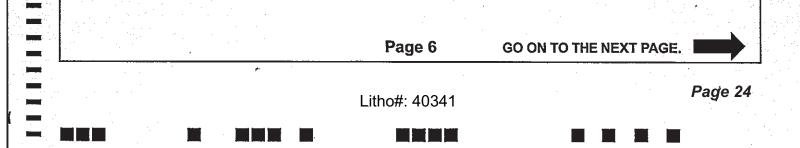
Both Harry and Jaden ate a portion of the same size pizza.

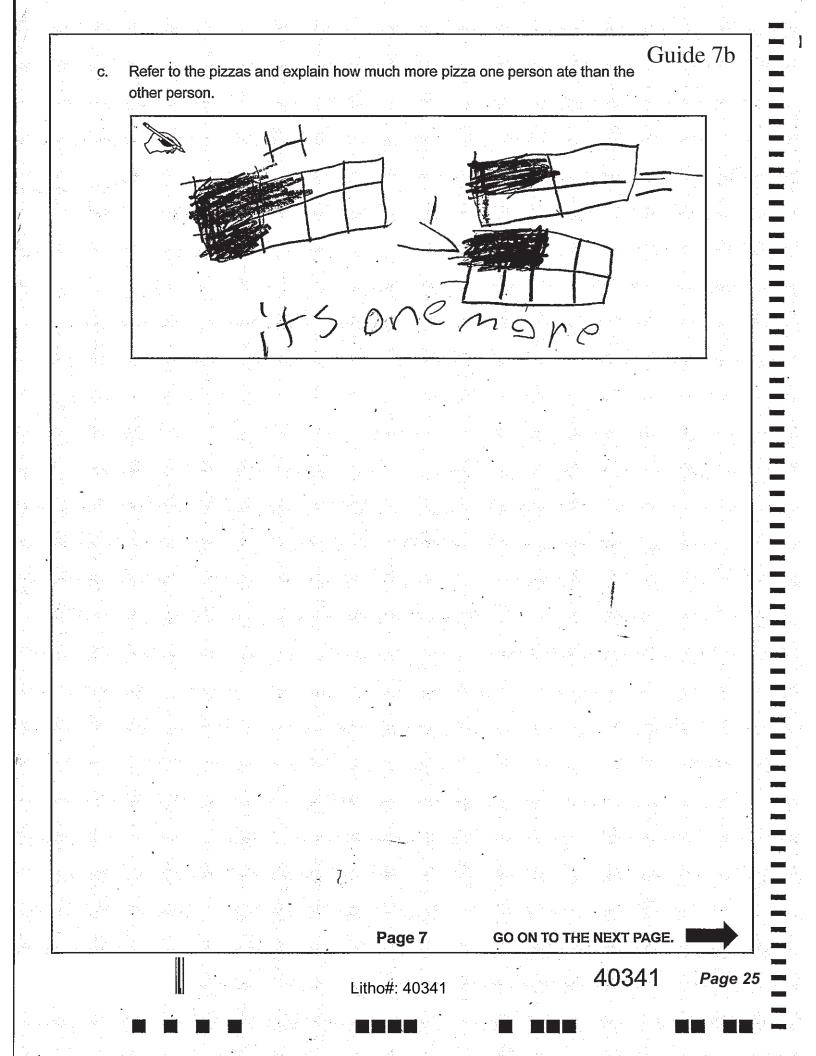
a. Show the portion of pizza that each student ate.



Use words and fractions to explain who ate the most pizza.







Guide 7Litho. 40341Total Content Points: 1(4.NF.2)Total Practice Points: 2(MP4, MP7)

The student indicates that Harry is the person who ate the most pizza and demonstrates her reasoning by showing that $\frac{2}{8}$ of Harry's pizza is equal to $\frac{1}{4}$ of Jaden's (4.NF.2). The student represents each amount with a diagram and indicates that Harry ate the most pizza, but does not specifically indicate how much more Harry ate. The statement "its one more" is too general, and there is no reference that the "one" is actually $\frac{1}{8}$ of a pizza (no credit for MP1). The student's diagrams show that fourths are double the size/portion of eighths (MP4). The student has accurate drawings and identifies $\frac{3}{8}$ as more pizza than $\frac{1}{4}$, but does not specify how much more pizza Harry ate than Jaden, nor clearly show any equations leading to the answer given (no credit for MP6). Work indicates that the student recognizes the two fractions need to be compared using like denominators by shading $\frac{2}{8}$ on one diagram and $\frac{1}{4}$ on the other, and by showing equivalency (MP7).

Total Awarded Points: 3 out of 5

Guide 8a Task 1. Pizza Task Both Harry and Jaden ate a portion of the same size pizza. Show the portion of pizza that each student ate. a. Harry's pizza: Jaden's pizza: Harry's pizza is cut into 8 pieces, Jaden's pizza is cut into 4 pieces, and he ate $\frac{3}{8}$ of the pizza. and she ate $\frac{1}{4}$ of her pizza. Use words and fractions to explain who ate the most pizza. b. Harry = 0.375 Harry ate Harry = 0.375 Harry ate the most Saden = 0.250 Pizza, Page 6 GO ON TO THE NEXT PAGE. Page 27 Litho#: 40066

Guide 8b Refer to the pizzas and explain how much more pizza one person ate than the C. other person.

Harry ate 0.125 Morie of the Pizza than Jaden.

Page 7

40066 Page 28

GO ON TO THE NEXT PAGE.

Litho#: 40066

Guide 8

Litho. 40066

Total Content Points: 0

Total Practice Points: 2 (MP1, MP4)

While this response identifies Harry as the person who ate the most pizza and provides accurate diagrams, the student converts $\frac{3}{8}$ and $\frac{1}{4}$ into decimals and completes the task using only decimals. Consequently, there is no indication that the student understands fractional relationships well enough to compare two fractions with unlike denominators (no credit for 4.NF.2). The student represents each amount with an accurate diagram, and identifies Harry as eating the most and specifies how much more was eaten (MP1). The student's diagrams show that fourths are double the size/portion of eighths (MP4). Without fractional math, precision and accuracy are not demonstrated as called for in this item. In addition, the student has converted the given fractions into decimals without showing their work in doing so, and the answer given in part c is also produced without any explanation or work shown (no credit for MP6). Although the response does arrive at correct answers to the task using decimals, the given work does not indicate the ability to find common denominators or compare the relative size of fractions (no credit for MP7).

Total Awarded Points: 2 out of 5

Guide 9a

Task 1. Pizza Task

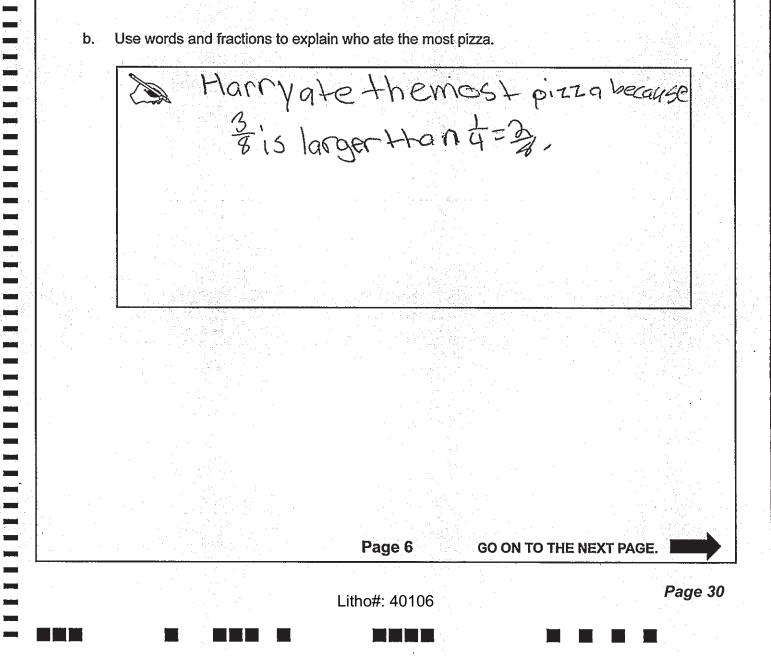
Both Harry and Jaden ate a portion of the same size pizza.

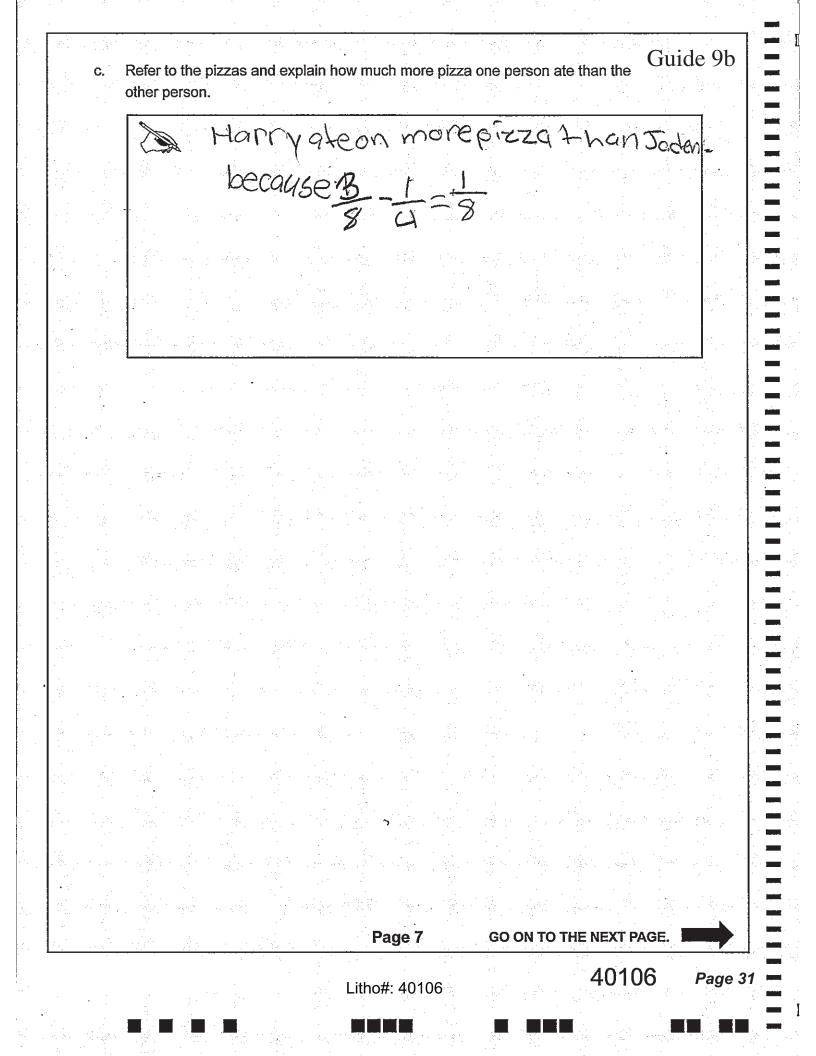
Show the portion of pizza that each student ate. a.

> Jaden's pizza: Harry's pizza: Harry's pizza is cut into 8 pieces, Jaden's pizza is cut into 4 pieces, and he ate $\frac{3}{8}$ of the pizza. and she ate $\frac{1}{4}$ of her pizza.

Use words and fractions to explain who ate the most pizza. b.

Harry gle themost pizza because Bis larger than 4=32,





Guide 9	Litho. 40106
Total Content Points: 1	(4.NF.2)
Total Practice Points: 1	(MP7)

This response identifies Harry as the person who ate the most pizza and explains by showing the equivalency of the fractions (" $\frac{3}{8}$ is larger than $\frac{1}{4} = \frac{2}{8}$ ") (4.NF.2). The student indicates that Harry ate the most pizza and specifies how much more was eaten, adequately responding to parts b and c. However, as the student never shows a representation of how much pizza each student ate, there is no response to part a (no credit for MP1). The student does not model the relationship between fourths and eighths in a response to part a, and although the answer to part b is comparing the sizes of the two given fractions, the student is not modeling the relationship mathematically (no credit for MP4). The student does not show a correct drawing in part a, and although the numeric answer in part c is correct, the accompanying statement "Harry ate on more pizza than Jaden" suggests a lack of full understanding of fractions (no credit for MP6). Work indicates that the student recognizes the two fractions can be compared using like

denominators ($\frac{1}{4} = \frac{2}{8}$) (MP7).

Total Awarded Points: 2 out of 5

Task 1. Pizza Task

Both Harry and Jaden ate a portion of the same size pizza.

a. Show the portion of pizza that each student ate.

Harry's pizza: Harry's pizza is cut into 8 pieces, and he ate $\frac{3}{8}$ of the pizza. Jaden's pizza is cut into 4 pieces, and she ate $\frac{1}{4}$ of her pizza.

b. Use words and fractions to explain who ate the most pizza.

So his pieces are going to be So his pieces are going to be Smalles But harrys pizza 15 ause the has ac th ty ale more 7 Page 6 GO ON TO THE NEXT PAGE Page 33 Litho#: 40002

Guide 10b Refer to the pizzas and explain how much more pizza one person ate than the C. other person. Sof the pizza and Jaden oing a fe j aint 31/2 Page 7 GO ON TO THE NEXT PAGE. 40002 Page 34 Litho#: 40002

Guide 10 Litho. 40002

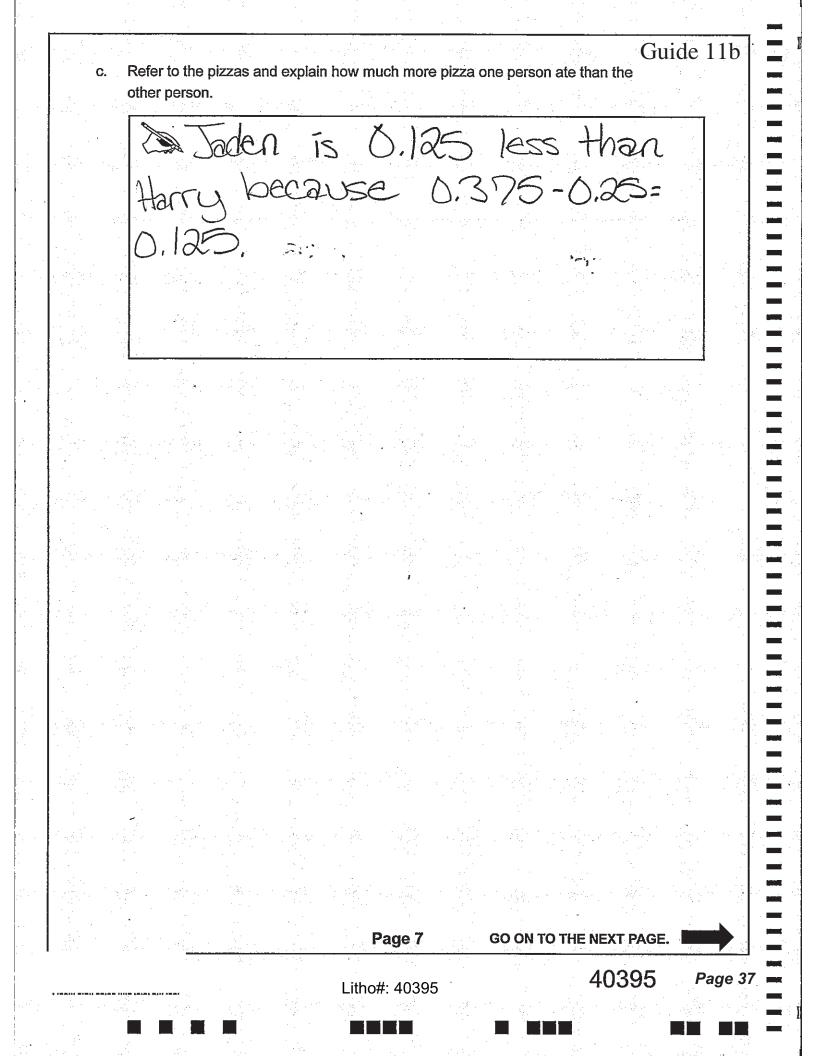
Total Content Points: 0

Total Practice Points: 1 (MP4)

This response identifies Harry as the person who ate the most pizza. However, without diagrams or a demonstration of equivalency with fractions, the student does not indicate a structural understanding of fractions (no credit for 4.NF.2). The student does not adequately respond to part a since there is no drawing, or to part c (no credit for MP1). The student provides a fraction notation indicating a comparison of the amounts (MP4). Despite providing a notation showing the fractional relationship, the student does not provide diagrams or indicate how much more pizza Harry ate (no credit for MP6). The student's work does not show how two fractions can be compared visually or through arithmetic by equalizing the denominators, so he does not clearly demonstrate an understanding of the structure of fractions (no credit for MP7).

Total Awarded Points: 1 out of 5

Guide 11a Task 1. Pizza Task Both Harry and Jaden ate a portion of the same size pizza. Show the portion of pizza that each student ate. а. Harry's pizza: Jaden's pizza: 0.25 0.375 Harry's pizza is cut into 8 pieces, Jaden's pizza is cut into 4 pieces, and he ate $\frac{3}{8}$ of the pizza. and she ate $\frac{1}{4}$ of her pizza. b. Use words and fractions to explain who ate the most pizza. you could change the Tractions into decimals, 3/2=0.375 and 1/4=0,25. Page 6 GO ON TO THE NEXT PAGE. Page 36 Litho#: 40395



Guide 11

Litho. 40395

Total Content Points: 0

Total Practice Points: 0

This response correctly identifies Harry as the person who ate the most pizza. The student converts $\frac{3}{8}$ and $\frac{1}{4}$ into decimals and completes the task using only decimals; consequently, the student has not clearly demonstrated that she understands fractions (no credit for 4.NF.2). The response indicates that Harry ate the most pizza and specifies how much more was eaten, but each amount is not represented with diagrams in part a (no credit for MP1). The student does not provide diagrams or a fraction notation to compare the amounts, so the response lacks appropriate modeling (no credit for MP4). Either an appropriate drawing or a correct fraction equation would be needed to demonstrate appropriate precision for the task given (no credit for MP6). Without using fractions to respond to the task, the student does not demonstrate an understanding of the structure of fractions (no credit for MP7).

Total Awarded Points: 0 out of 5

Task 1. Pizza Task

b.

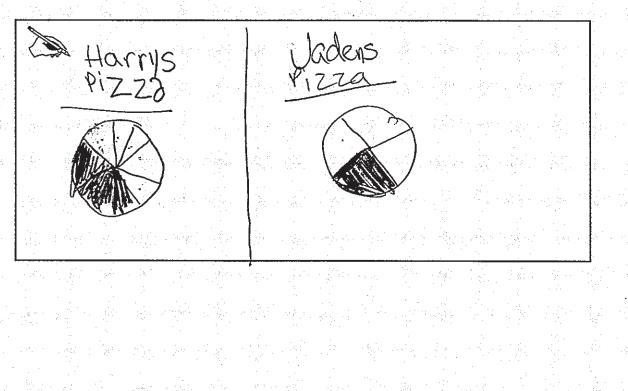
Both Harry and Jaden ate a portion of the same size pizza.

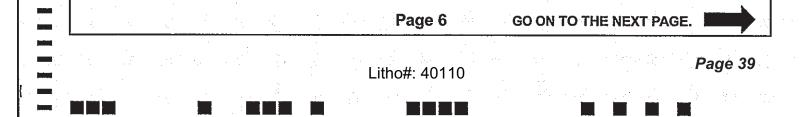
Guide 12a

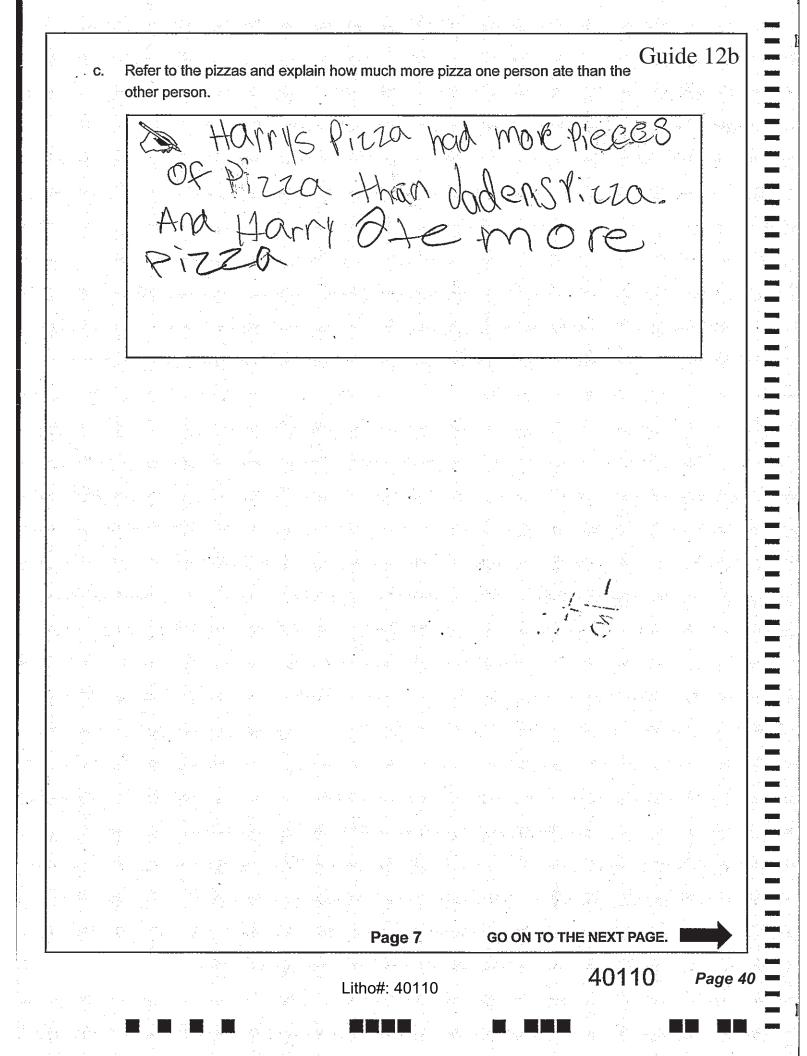
a. Show the portion of pizza that each student ate.

, Harry's pizza:	Jaden's pizza:
Harry's pizza is cut into 8 pieces, and he ate $\frac{3}{8}$ of the pizza.	Jaden's pizza is cut into 4 pieces, and she ate $\frac{1}{4}$ of her pizza.
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Use words and fractions to explain who ate the most pizza.







Guide 12

Litho. 40110

Total Content Points: 0

Total Practice Points: 0

While this response correctly identifies Harry as the person who ate the most pizza and has correct diagrams, there is no clear indication that the student understands fractional relationships (no credit for 4.NF.2). The student attempts a drawing to illustrate the problem, but does not find how much more pizza Harry ate than Jaden (no credit for MP1). Although the student does complete drawings, the size of the slices of Harry's

pizza is inconsistent, and the drawing shows $\frac{3}{8}$ as equal to $\frac{1}{2}$ (no credit for MP4). The

flawed drawing and the confusing explanation in part c that implies Harry ate more pizza because he began with more pieces demonstrate a lack of precision (no credit for MP6). The work presented does not indicate that the student recognizes the two fractions need to be compared using like denominators, as there is no clear statement that like denominators are needed and the drawing is not clear and precise enough to illustrate the comparison (no credit for MP7).

Total Awarded Points: 0 out of 5