

Tennessee Comprehensive Assessment Program / Mathematics

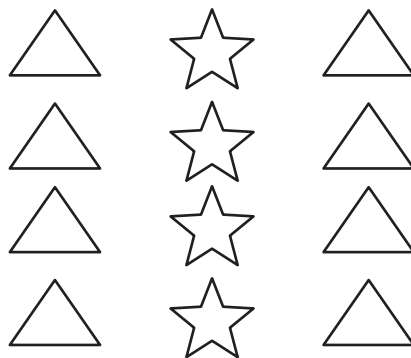
# TCAP/CRA PILOT 2012



## Task 3 : Triangles and Stars

### Scoring Guide

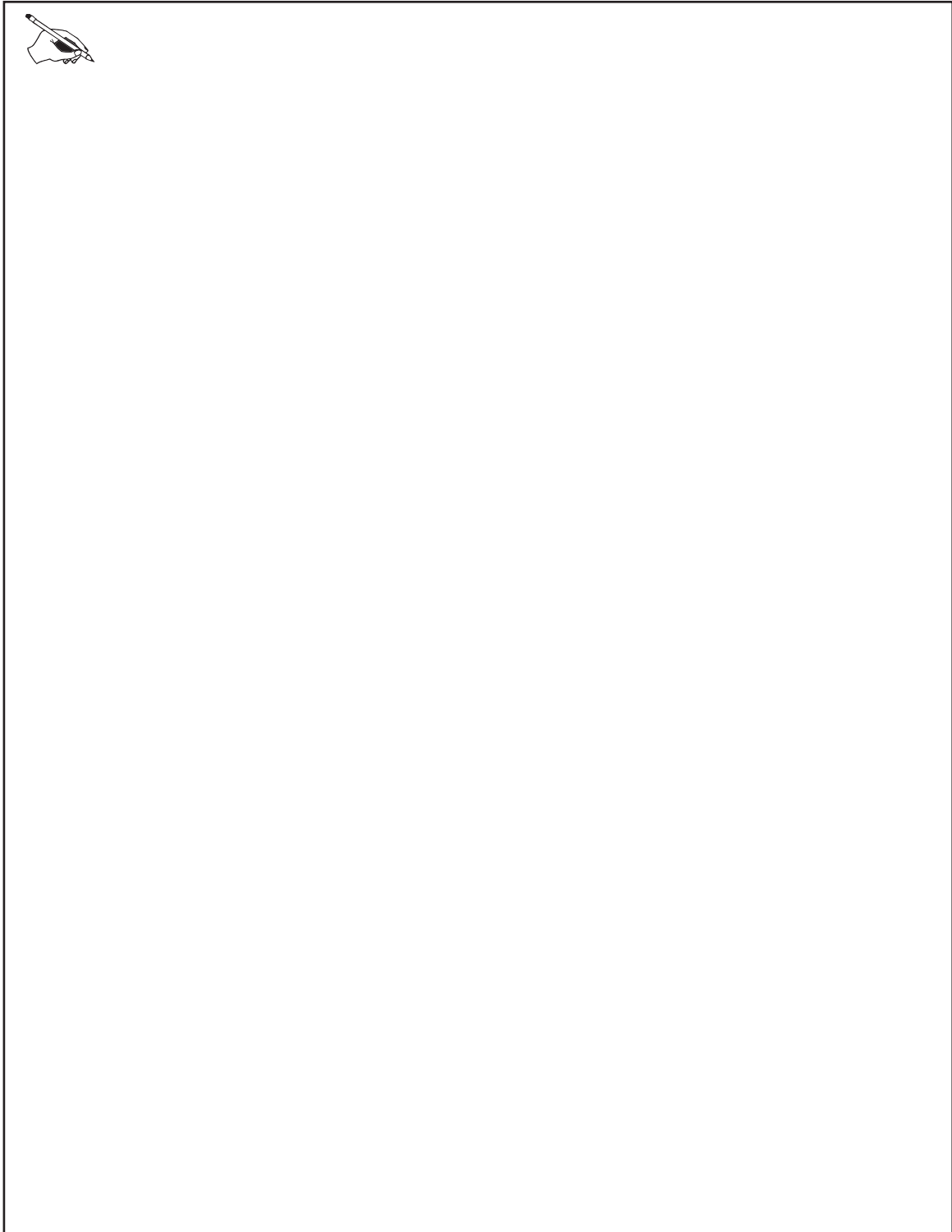
### Task 3. Triangles and Stars Task



- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

Handwriting area for the student's response.

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.



### 3. Triangles and Stars Task Scoring Guide

#### The CCSS for Mathematical Content (3 points)

- 6.RP.1 Uses ratios and/or ratio language in responding to the question; writes three equivalent ratios to describe the ratio of triangles to stars in the picture. \_\_\_\_\_
- 6.RP.2 Correctly determines a unit rate associated with the problem, e.g., 2 triangles per star or  $\frac{1}{2}$  star per triangle. \_\_\_\_\_
- 6.RP.3b Interprets unit rate appropriately to determine the number of triangles when there are 20 stars, e.g., there are twice as many triangles as stars, so  $20 \times 2 = 40$ . \_\_\_\_\_

**Total Content Points** \_\_\_\_\_

#### The CCSS for Mathematical Practices (4 points)

- MP1 Recognizes proportionality in part b; completes both parts of the task. \_\_\_\_\_  
(MP1: Make sense of problems and persevere in solving them.)
- MP2 Abstracts the values from the picture, forms equivalent ratios, and uses multiplicative process on those numbers to solve the problem. Re-contextualizes the results correctly. \_\_\_\_\_  
(MP2: Reason abstractly and quantitatively.)
- MP6 Accurately refers to the diagram, writes correct equations, and labels quantities in both questions correctly. \_\_\_\_\_  
(MP6: Attend to precision.)
- MP7 Work indicates that the student understands the proportional relationship between the triangles and stars indicated by the language and the multiplicative relationship that is therefore implied. \_\_\_\_\_  
(MP7: Look for and make use of structure.)

**Total Practice Points** \_\_\_\_\_

**Total Awarded Points** \_\_\_\_\_

## The CCSS for Mathematical Content Addressed In This Task

### Understand ratio concepts and use ratio reasoning to solve problems.

- 6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2: 1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*
- 6.RP.2 Understand the concept of a unit rate  $a/b$  associated with a ratio  $a: b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $3/4$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”*

### Understand ratio concepts and use ratio reasoning to solve problems.

**Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.**

- 6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

## The CCSS for Mathematical Practices\*

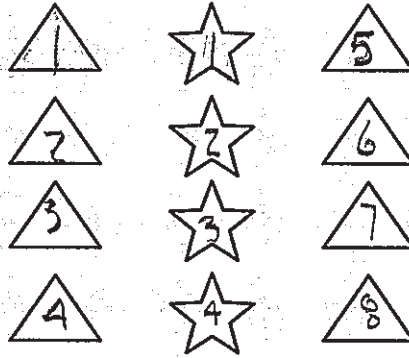
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

\*Gray text indicates Mathematical Practices not addressed in this task.

Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

Task 3. Triangles and Stars Task

Guide 1a



- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

Handwritten student work showing a ratio table, a simplified ratio, and a written explanation.

Handwritten text: "which means that there are twice as many triangles as stars."

triangles	stars
8	4
4	2
2	1


Handwritten ratio:  $\frac{8}{4} = \frac{4}{2} = \frac{2}{1}$

Handwritten division work:

$$\begin{array}{r} 2 \overline{) 8} \\ \underline{4} \phantom{0} \\ 4 \phantom{0} \\ \underline{4} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 4} \\ \underline{4} \\ 0 \end{array}$$

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 since there are twice as many triangles as stars you have to multiply  $20 \times 2 = 40$ , or

$$\frac{20}{1} \times \frac{1}{2} = 40$$

40 triangles

T	S
40	20
20	10
2	1

T	S
40	20
4	2
2	1

T	S
40	20
8	4
4	2
2	1

Guide 1

Litho 60368

Total Content Points: 3 (6.RP.1, 6.RP.2, 6.RP.3b)

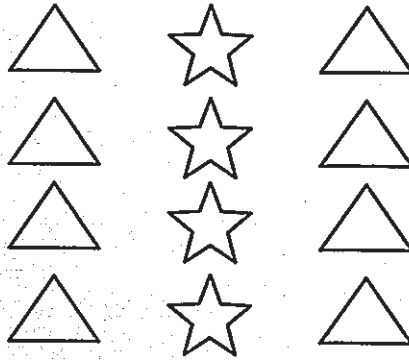
Total Practice Points: 4 (MP1, MP2, MP6, MP7)

The student provides three equivalent ratios to describe the proportion of triangles to stars in the picture (6.RP.1), one of which is the unit rate (6.RP.2). The student interprets the unit rate appropriately to determine that there will be 40 triangles when there are 20 stars (6.RP.3b). The student correctly recognizes the proportion of stars to triangles in part b (MP1) and forms equivalent ratios, solves the problem by multiplying the unit rate by the number of stars, and correctly states that there are 40 triangles for every 20 stars (MP2). The student shows precision in correctly writing and labeling equations and the quantities of stars and triangles (MP6) and indicates that she understands the proportional relationship between the triangles and stars and the multiplicative relationship this represents (MP7).


Total Awarded Points: 7 out of 7




Task 3. Triangles and Stars Task



- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

 8:4, 4:2, 2:1

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 There are 4 stars right now, if there were 20 stars, that's  $4 \cdot 5 = 20$ ; there are 8 triangles, so you multiply that by 5 which gives you  $8 \cdot 5 = 40$ . That means when there are 20 stars, there are 40 triangles. You can also see that the ratio of triangle to stars is 2:1, so you could multiply  $20 \cdot 2 = 40$ .

Guide 2

Litho 60144

Total Content Points: 3 (6.RP.1, 6.RP.2, 6.RP.3b)

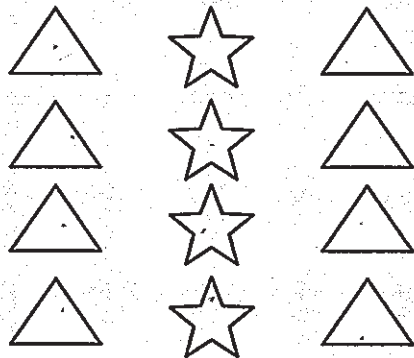
Total Practice Points: 4 (MP1, MP2, MP6, MP7)

This response provides three equivalent ratios to describe the proportion of triangles to stars in the picture (6.RP.1), one of which is the unit rate (6.RP.2). The student uses the unit rate as one of his two methods to find the number of triangles there would be if there were 20 stars (6.RP.3b). The student correctly recognizes the proportion of stars to triangles in part b (MP1) and forms equivalent ratios, solves the problem by multiplying the unit rate by the number of stars, and correctly states that there are 40 triangles for every 20 stars (MP2). The student shows precision in correctly writing and labeling equations and the quantities of stars and triangles (MP6), and the explanation in part b indicates he understands both the proportional relationship between the triangles and stars and the multiplicative relationship this represents (MP7).

Total Awarded Points: 7 out of 7

Task 3. Triangles and Stars Task

Guide 3a




$$\frac{4 \times 2}{12 \times 2} = \frac{8}{24}$$

$$\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

$$\frac{2}{3} \quad \frac{8 \times 2}{12 \times 2} = \frac{16}{24}$$


- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

  $\frac{8}{12}$  to  $\frac{4}{12}$     $\frac{2}{3}$  to  $\frac{1}{3}$

Unit Rate

Number of triangles	2	4	6	8
Number of Stars	1x2	2x2	3x2	4x2

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 To use the unit rate to find out how many triangles there will be when there are 20 stars, you need to multiply the number of stars by 2 to get the number of triangles.  $20 \times 2 = 40$  so, there would be 40 triangles.

Triangles	40
Stars	$20 \times 2$

Guide 3

Litho 60015

Total Content Points: 3 (6.RP.1, 6.RP.2, 6.RP.3)

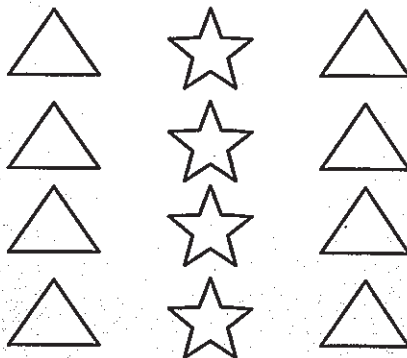
Total Practice Points: 3 (MP1, MP2, MP7)

The student provides four equivalent ratios of triangles to stars in the picture (6.RP.1), one of which is the unit rate (6.RP.2). The student explains how to use the unit rate appropriately to determine the number of triangles when there are 20 stars (6.RP.3). The student correctly recognizes the proportion of stars to triangles in part b (MP1) then forms equivalent ratios, solves the problem by multiplying the unit rate by the number of stars, and correctly states that there are 40 triangles for every star (MP 2). The student demonstrates that she understands the proportional relationship between the triangles and stars and the multiplicative relationship that relationship represents (MP7). The student's charts are not entirely correct, as the multiplication shown (as an explanation of how to derive the number of triangles from the number of stars) shows a lack of attention to precision (no credit for MP6).


Total Awarded Points: 6 out of 7

Task 3. Triangles and Stars Task


Guide 4a



- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

  $8:4$   
 $4:2$   
 $2:1$

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 40 triangles

$$20 \times 2 = 40$$
$$20 \div 4 = 5$$
$$5 \times 8 = 40$$



Guide 4

Litho 60081

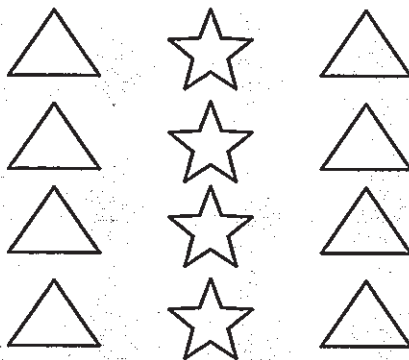
Total Content Points: 3 (6.RP.1, 6.RP.2, 6.RP.3b)

Total Practice Points: 3 (MP1, MP2, MP7)


The student provides three equivalent ratios of triangles to stars in the picture (6.RP.1), one of which is the unit rate (6.RP.2), and interprets the unit rate appropriately to determine the number of triangles when there are 20 stars (6.RP.3b). The student correctly recognizes the proportion of stars to triangles in part b (MP1), solves the problem by multiplying the unit rate by the number of stars, and correctly states that there are 40 triangles for every 20 stars (MP2). The response demonstrates that the student understands the proportional relationship between the triangles and stars and the multiplicative relationship this represents (MP7). The student's response lacks labeling throughout, and his explanation in part b makes insufficient reference to the diagram (no credit for MP6).

Total Awarded Points: 6 out of 7

Task 3. Triangles and Stars Task



- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

 Triangles: Stars — 8:4 / 2:1  
Unit rate: 2 triangles per 1 star

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.



If the ratio is 2:1, and there are 20 stars, you would multiply  $20 \times 2 = 40$

there are 40 triangles if there are 20 stars.



Guide 5

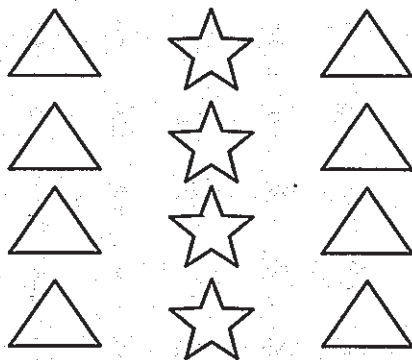
Litho 60333

Total Content Points: 2 (6.RP.2, 6.RP.3b)


Total Practice Points: 3 (MP2, MP6, MP7)

The student correctly determines a unit rate (6.RP.2) and interprets the unit rate appropriately to determine the number of triangles when there are 20 stars (6.RP.3b). The student only provides two equivalent ratios to describe the ratio of triangles to stars in the picture (no credit for 6.RP.1). The student therefore does not complete both parts of the task (no credit for MP1). The student solves the problem by multiplying the unit rate by the number of stars, and correctly states that there are 40 triangles for every 20 stars (MP2). The response demonstrates that the student understands the proportional relationship between the triangles and stars and the multiplicative relationship this represents by scaling up the ratio in part b (MP7). The student's equation and ratios are correct, with the values arrived at properly labeled (MP6).

Total Awarded Points: 5 out of 7




- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

 There are 8 triangles and 4 stars, so there are 2 triangles for every star. Two other ratios are 2:1 or 8:4. Ratios can be written three ways, all of which are listed here.

$\frac{8}{4} = \frac{2\triangle}{1\star}$  so

$\frac{8}{4}$  8:4 8 to 4  
or  
 $\frac{2}{1}$  2:1 2 to 1

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 You increase the number of stars by 500%, so you should do the same to the triangles. Eight times five is 40.

$$\frac{8}{4} \cdot \frac{5}{1} = \frac{40}{20}$$
$$8 \cdot 5 = 40$$
$$\frac{20}{4} = 5$$

Guide 6

Litho 60059

Total Content Points: 2 (6.RP.2, 6.RP.3b)

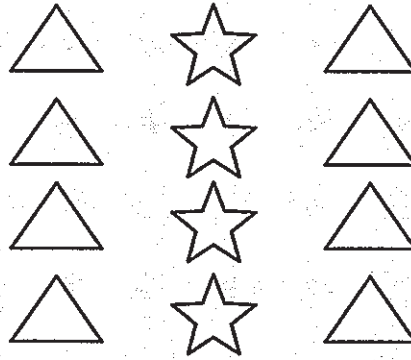
Total Practice Points: 3 (MP2, MP6, MP7)

The student correctly determines a unit rate (6.RP.2). She only provides two ratios (no credit for 6.RP.1), although she interprets unit rate appropriately to find the correct number of triangles when there are 20 stars (6.RP.3b). The student has not fully completed both parts of the task (no credit for MP1). The student's calculations do not use the unit rate, but demonstrate that the student abstracted values from the picture, formed equivalent ratios, and re-contextualized the results (MP2). The equations are correct and the explanations given serve to label the resulting quantities (MP6). The student uses multiplication to solve the problem, which demonstrates an understanding of the relationship between proportions and multiplication (MP7).


Total Awarded Points: 5 out of 7

Task 3. Triangles and Stars Task

Guide 7a




- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

  $\frac{8}{4} = \frac{16}{8} = \frac{32}{16}$



- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.


$$\frac{8}{4} = \frac{x}{20} = 40$$
$$20 \times 8 = 160 \div 4 = 40$$

Guide 7

Litho 60134

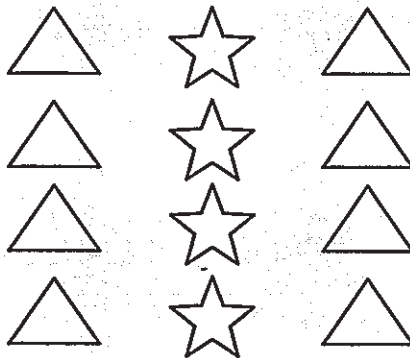
Total Content Points: 2 (6.RP.1, 6.RP.3b)

Total Practice Points: 2 (MP2, MP7)


This response provides three equivalent ratios that describe the ratios of triangles to stars in the picture (6.RP.1). No unit rate is provided (no credit for 6.RP.2). The student correctly interprets the unit rate to create ratios; these are used to determine there will be 40 triangles when there are 20 stars (6.RP.3b). The student does not correctly complete both parts of the task (no credit for MP1). The student's calculations, showing how the correct ratio of triangles to stars was achieved, demonstrate that the student abstracted values from the picture, formed equivalent ratios, and re-contextualized the results (MP2). The student uses multiplication to solve the problem, and indicates an understanding of the relationship between ratios and multiplication (MP7). The student shows a lack of attention to precision in the incorrect equations shown in part b (no credit for MP6).

Total Awarded Points: 4 out of 7

Task 3. Triangles and Stars Task




- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

 8:4      8 to 4

$\frac{8}{4}$

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 Since there are 4 stars and together to twenty you would have to multiply it by 5, so you would have to multiply 8 by 5. Then you would get 40, so it would look like this.

$$\frac{40 \text{ triangles}}{20 \text{ stars}}$$

Guide 8

Litho 60033

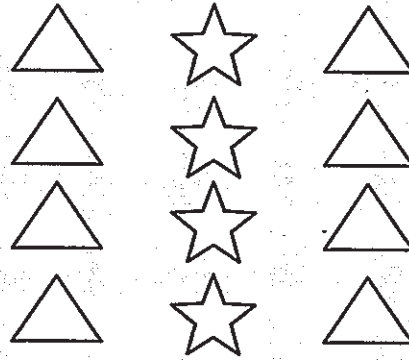
Total Content Points: 1 (6.RP.3b)

Total Practice Points: 2 (MP2, MP7)

The student shows three different methods of stating one ratio, rather than three equivalent ratios or a unit rate (no credit for 6.RP.1, no credit for 6.RP.2). The student interprets the unit rate appropriately to find the correct number of triangles there will be when there are 20 stars (6.RP.3b). The student has not correctly completed both parts of the task (no credit for MP1). The student uses multiplication to solve the problem (MP7), and he abstracts values from the picture, forms equivalent ratios, and re-contextualizes the results (MP2). Although the student explains his process in part b, the explanation lacks completeness and precision, and the student does not present an equation (no credit for MP6).

Total Awarded Points: 3 out of 7

Task 3. Triangles and Stars Task



- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.


$\frac{4}{12}$  stars |  $\frac{2}{6}$  stars |  $\frac{1}{3}$  stars

In order to get this answer you have to simplify

$\frac{4}{12}$  3 times

$4 \div 2 = 2$   $2 \div 2 = 1$   
 $12 \div 2 = 6$   $6 \div 2 = 3$

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 If there are 20 stars then the ratio of stars has just been multiplied by 5 so multiply 8 by 5 and you get 40. So your answer is there are 40 triangles.

Guide 9

Litho 60357

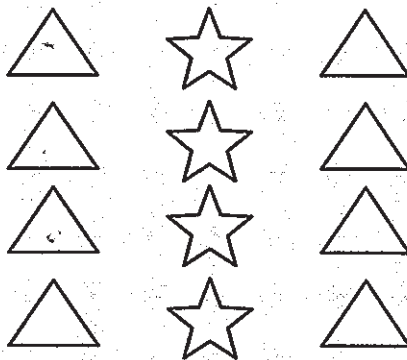
Total Content Points: 1 (6.RP.3b)

Total Practice Points: 2 (MP2, MP7)

The student does not provide equivalent ratios of triangles to stars or a unit rate (no credit for 6.RP.1, no credit for 6.RP.2). However, the student interprets unit rate appropriately and determines that there will be 40 triangles when there are 20 stars (6.RP.3b). The student does not complete both parts of the task (no credit for MP1). The use of calculations in part b demonstrates that the student abstracted values from the picture, formed equivalent ratios, and re-contextualized the results (MP2). The response uses multiplication to solve the problem (MP7). The incorrect answers and calculations in part a indicate a lack of attention to precision (no credit for MP6).

Total Awarded Points: 3 out of 7





- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

*You count the amount of stars, then you count the total number of shapes which is*

*4 Stars =  $\frac{4}{12}$  =  $\frac{1}{3}$*

*12 total shapes = 6 =  $\frac{1}{3}$*

*Or you can count the number of stars and just count the number of triangles. Then turn them into a fraction and simplify.*


*4 Stars*


*8 triangles*

*$\frac{1}{2}$  stars*

*$\frac{2}{1}$  triangles*

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 You can divided the total number of all shapes to the number of stars. Since the triangles have the ratio as the stars then they will be the same. Add the stars and the triangles ratios up and that is the total number of shapes. Then simplify the total of ~~triangles~~ triangles to the total of shapes.




$$= 40 \text{ total shapes}$$

$$\div 20 \text{ stars}$$

$$\frac{20 \text{ triangles}}{40 \text{ total or shapes}} = \frac{10}{20} = \frac{5}{10} = \frac{1}{2}$$

Simplified answer



Guide 10

Litho 60069

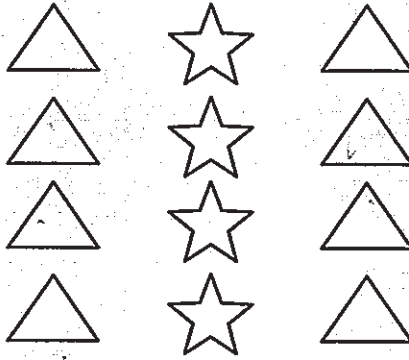
Total Content Points: 1 (6.RP.2)

Total Practice Points: 0


The student correctly identifies a unit rate that describes the ratio of stars to triangles (6.RP.2). The student does not use the unit rate to correctly determine the ratio between the number of triangles and stars (no credit for 6.RP.3b), nor are three equivalent ratios provided (no credit for 6.NF.1). The student does not correctly complete both parts of the task (no credit for MP1). The student incorrectly interprets the values from the picture in part b (no credit for MP2). The work shown does not demonstrate an understanding of the multiplicative aspect of ratios (no credit for MP7). The confusing explanation and incorrect expression in part b indicate a lack of attention to precision (no credit for MP6).

Total Awarded Points: 1 out of 7


Task 3. Triangles and Stars Task



- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

 8:4  
 16:8  
 4:2

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 20:15  
If you simplify down to 4:3  
then multiply it by three the  
ratio will be (20 triangles) to  
(15 stars)

Guide 11

Litho 60017

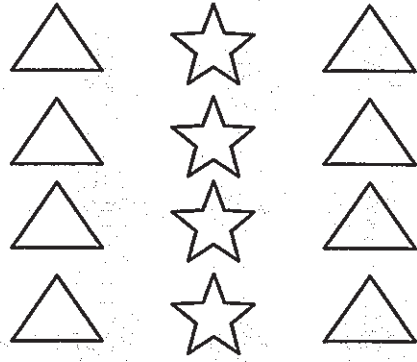
Total Content Points: 1 (6.RP.1)

Total Practice Points: 0


The student correctly identifies three equivalent ratios in part a (6.RP.1). The student does not identify a unit rate (no credit for 6.RP.2), use a unit rate to determine the number of triangles when there are 20 stars (no credit for 6.RP.3b), or complete both parts of the task (MP1). The student has not used multiplicative processes on numbers abstracted from the picture to solve the problem (no credit for MP2) and the explanation given in part b does not show understanding of the relationship between proportions and multiplication (no credit for MP7). The student's confusing explanation and incorrect multiplication in part b indicate a lack of precision (no credit for MP6).

Total Awarded Points: 1 out of 7


Task 3. Triangles and Stars Task



- a. Write a unit rate and two other, different ratios, all of which accurately describe the ratio of triangles to stars in the picture above.

  $\frac{4}{8}, \frac{1}{2}, 12$  ?

- b. If the ratio of triangles to stars stays the same, explain in words and equations how to use the unit rate to find the number of triangles there will be when there are 20 stars in the picture.

 You would add 20 to 12





Total Content Points: 0

Total Practice Points: 0

This response does not provide equivalent ratios or a unit rate. The student does not identify a unit rate or demonstrate knowledge of the use of proportionality to correctly determine the ratio between the number of triangles and stars. Because the given  $\frac{1}{2}$  is not labeled, it cannot be assumed to be a correct ratio of stars to triangles, and the student is not given credit for finding a unit rate (no credit for 6.RP.1, no credit for 6.RP.2, no credit for 6.RP.3b). The student has not completed either task (no credit for MP1), has not used or indicated understanding of the multiplicative process (no credit for MP2, no credit for MP7), and shows no evidence of attention to precision due to a lack of labeling and clear explanation (no credit for MP6).

Total Awarded Points: 0 out of 7