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Arc Overview

In this set of related math tasks, 3rd Grade students will solve word problems using multiplication and division within 100. They will move toward using algebraic reasoning involving multiplication and division equations with unknowns.

The Arc Preview table on page 4 provides all of the task questions contained in this arc. The tasks are aligned to standards 3.OA.A.2, 3.OA.A.3, 3.OA.A.4 and 3.OA.B.6.

- Task 1 will develop understanding of the relationship between multiplication and division using array models.
- Task 2 will explore writing mathematical explanations to solve a real world problem involving multiplication and division.
- Task 3 will solidify understanding multiplication and division within 100.
- Task 4 will introduce the use unknowns in multiplication and division equations.
- Task 5 will develop the use of equations with unknowns in multiplication and division equations.
- Task 6 will solidify understanding of multiplication and division, including the use of equations with unknowns.

Note that the some of the Essential Understandings listed in each task were modified from those contained in Pearson's EnVision Math series. Others were taken from NCTM's Developing Essential Understanding series. Tennessee State Mathematics Standards were retrieved from http://www.tn.gov/education/standards/math.shtml.

By the end of these six tasks, students will be able to answer the following overarching questions:

- How can you use arrays to model multiplication and division?
- How can division be used to find how many equal groups or how many are in each group?
- What are some different ways to show mathematical explanations or reasoning?
- What are some examples of real world problems involving joining or separating equal groups?
- How can multiplication and division be used as inverse operations to find division and multiplication facts?
- How can unknowns be used in multiplication and division equations to help you solve joining or separating problems?

The assessing questions, advancing questions, and whole group questions provided in this guide will ensure that students are working in ways aligned to the Standards for Mathematical Practice. Although the students will not be aware that this is occurring, the teacher can guide the process so that each MP (Mathematical Practice) is covered through good explanations, understanding of context, and clarification of reasoning behind solutions.

Arc Preview

Task 1: Kitchen Floor Tiling

Miranda wants to use 18 tiles to put in a new floor for her kitchen.

- a) Draw as many different arrays as you can, using all 18 tiles. Write a multiplication and division equation to represent each array.
- b) What are all the factors of 18? How can you be sure these are the only factors of 18?

Goals for Task 1:

- Show a whole number partitioned into equal groups.
- Use multiplication and division within 100 involving equal groups.

Standards for Task 1:

3.OA.A.2 Interpret whole number quotients of whole numbers, e.g., interpret 56÷8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects.

3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Task 2: Soccer Goals

Aaron is making a picture graph. Each picture of a soccer ball stands for 4 goals he scored for his team. The row for January has 6 soccer balls.

- a) How many goals did Aaron score during January? Write a multiplication equation to show how you solved for the answer.
- b) If he scores 48 goals in February, how many soccer balls should he put on the graph? How do you know?

Goals for Task 2:

- Interpret whole number quotients of whole numbers.
- Use multiplication and division within 100 to solve word problems.

Standards for Task 2:

3.OA.A.2 Interpret whole number quotients of whole numbers, e.g., interpret 56÷8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects.

3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Task 3: Name that Number

Solidifying Understanding

- 1. Mia bought 2 copies of the same book. She spent \$18. What was the cost of one book? Write an equation and explain your reasoning.
- 2. Mr. Martin buys 36 biscuits for a class breakfast. He places them on plates for his students. If he places 12 biscuits on each plate, how many plates does he need? Write an equation and explain your reasoning.

Standards for Task 3:

division problem.

involving equal groups.

Goals for Task 3:

3.OA.A.2 Interpret whole number quotients of whole numbers, e.g., interpret 56÷8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects.

Use multiplication and division within 100

Write an equation to solve a multiplication or

3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Task 4: Surprise Birthday Party

Miranda's parents bought 24 balloons for Miranda and some friends to share at her birthday party.

- a) If she and 5 friends share the balloons, how many will each child receive? Write an equation and explain your reasoning.
- b) Miranda decides instead to make bundles of 3 balloons to tie around the room. How many bundles would she be able to make?

Goals for Task 4:

- Determine the unknown number in multiplication and division equations.
- Relate three whole numbers in multiplication and division.

Standards for Task 4:

- **3.OA.A.2** Interpret whole number quotients of whole numbers, e.g., interpret 56÷8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects.
- **3.OA.A.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$ **3.OA.B.6** Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

Task 5: Pin the Number on the Chart

Miranda's mother needs to get some supplies for the surprise birthday party for her daughter. Help her complete the supply list based on the information below.

BIRTHDAY SUPPLY LIST

| Item | Cost per Pack | Number of packs purchased | Total Cost |
|---------|---------------|---------------------------|------------|
| Bowls | \$4 | | |
| Cups | \$3 | | \$9 |
| Napkins | | 2 | \$6 |

- a) What is the cost of each pack of napkins if she spent \$6 buying 2 packs of napkins?
 Solve by writing an equation with an unknown and fill in the chart.
- b) If each pack of cups cost \$3, how many packs can she buy with \$9? Solve by writing an equation with an unknown and fill in the chart.
- c) There are 6 bowls in one pack and 18 napkins in one pack. If Miranda's mother has 2 packs of napkins, how many packs of bowls will she need to buy if she wants to have an equal amount of bowls and napkins? Solve by writing an equation with an unknown and fill in the chart.
- d) What will be the total cost of bowls? Solve by writing an equation with an unknown and fill in the chart.

Goals for Task 5:

- Determine the unknown number in multiplication and division equations.
- Relate three whole numbers in multiplication and division.

Standards for Task 5:

- **3.OA.A.2** Interpret whole number quotients of whole numbers, e.g., interpret 56÷8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects.
- **3.OA.A.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$ **3.OA.B.6** Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

Task 6: Musical Chairs

Miranda has 5 tables.

- a) If 30 people will be attending her party, how many chairs should be at each table to form equal groups? Solve by writing an equation with an unknown.
- b) If Miranda has 45 flowers and she wants to put a vase with the same number of flowers on each table, how many flowers should be in each vase? Solve by writing an equation with an unknown.

Goals for Task 6:

- Determine the unknown number in multiplication and division equations.
- Relate three whole numbers in multiplication and division.

Standards for Task 6:

- **3.OA.A.2** Interpret whole number quotients of whole numbers, e.g., interpret 56÷8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects.
- **3.OA.A.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$ **3.OA.B.6** Understand division as an unknownfactor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

Tasks' Standards Alignment

| Task | 3.0A.A.2 | 3.0A.A.3 | 3.0A.A.4 | 3.0A.B.6 | MP 1 | MP 2 | MP 3 | MP 4 | MP 5 | MP 6 | MP 7 | MP 8 |
|---|----------|----------|----------|----------|------|------|------|------|------|------|------|------|
| Task 1 Kitchen Floor Tiling | ~ | • | | | ~ | • | • | ~ | ~ | • | ~ | |
| Task 2 Making a Picture Graph | ~ | ~ | | | ~ | ~ | ~ | ~ | • | • | ~ | |
| Task 3 Name That Number Solidifying Understanding | ~ | • | | | • | • | • | • | • | • | • | |
| Task 4 Surprise Birthday Party | ~ | • | ~ | • | • | • | • | ~ | • | • | ~ | |
| Task 5 Pin the Number on the Chart | ~ | • | ~ | • | ~ | • | • | • | ~ | • | ~ | ~ |
| Task 6 Musical Chairs Solidifying Understanding | V | • | • | • | • | • | • | • | • | • | • | ~ |

The Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

| Name | | | | | | | | |
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| | | | | | | | | |

Task 1: Kitchen Floor Tiling

Miranda wants to use 18 tiles to put in a new floor for her kitchen.

a) Draw as many different arrays as you can, using all 18 tiles. Write a multiplication and division equation to represent each array.

b) What are all the factors of 18? How can you be sure these are the only factors of 18?

Task 1

Tennessee Department of Education: Lesson Guide 1

Task 1: Kitchen Floor Tiling

3rd Grade

Miranda wants to use 18 tiles to put in a new floor for her kitchen.

- a) Draw as many different arrays as you can, using all 18 tiles. Write a multiplication and division equation to represent each array.
- b) What are all the factors of 18? How can you be sure these are the only factors of 18?

Teacher Notes:

This lesson allows the teacher to emphasize various ways to teach students how to organize all possible factor combinations. In this task a 2×9 array is the same thing as a 9×2 array since the focus is on the factors of 18.

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|--|--|
| Tennessee State Standards for Mathematical | Tennessee State Standards for Mathematical |
| Content | Practice |
| 3.OA.A.2 Interpret whole number quotients of whole | 1. Make sense of problems and persevere in |
| numbers, e.g., interpret 56÷8 as the number of | solving them. |
| objects in each share when 56 objects are partitioned | 2. Reason abstractly and quantitatively. |
| equally into 8 shares, or as a number of shares when | 3. Construct viable arguments and critique the |
| 56 objects are partitioned into equal shares of 8 | reasoning of others. |
| objects. | 4. Model with mathematics. |
| 3.OA.A.3 Use multiplication and division within 100 | 5. Use appropriate tools strategically. |
| to solve word problems in situations involving equal | 6. Attend to precision. |
| groups, arrays, and measurement quantities, e.g., by | 7. Look for and make use of structure. |
| using drawings and equations with a symbol for the | 8. Look for and express regularity in repeated |
| unknown number to represent the problem. | reasoning. |
| | |

Essential Understandings:

- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.
- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- An array involves joining equal groups and is one way to think about multiplication and division.
- Multiplication can be used to find the total number of objects when there are a specific number of groups with the same number of objects.

Explore Phase

Possible Solution Paths Assessing and Advancing Questions Assessing Questions: a) Students show arrays that include the fact family Can you explain your work to another student to combinations of the following: show how you solved for your arrays? 1) 1x18=18, 18x1=18, 18÷1=18, 18÷18=1, showing How did you know how to begin? one row or column of 18 What does the factor 3 represent in your array? Using your arrays, can you tell me how many factors 18 has? What are they? • Can you show me how the factors relate to the 2) 2x9=18, 9x2=18, $18\div 2=9$, $18\div 9=2$, showing two rows of 9 or nine rows of 2 array? Can you explain what division means? Can you explain how 2x9 is different from 2+9? **Advancing Questions:** What is an array? 3) 3x6=18, 6x3=18, $18\div 3=6$, $18\div 6=3$ showing three What repeated addition sentences can be used rows of 8 or eight rows of 3 to add up to 18?

Task 1 10

| | What does the word "factor" mean? Can you name any factors of 18? | | | | |
|--|--|--|--|--|--|
| Arrays could be transposed. | | | | | |
| b) Factors: 1,2,3,6,9,18 Students' explanations will vary. One possible explanation is that when looking for factors systematically starting with the number 1, students will report 1 x18, then 2 x 9, then 3 x 6, and note that 4 and 5 are not factors. When they reach 6, they notice it is a repeat pair and can know they are done. | Assessing Questions: Can you tell me about the different arrays of tiles you have? What array did you begin with? Why? How did you decide which array to draw next? Advancing Questions: Is 1 a factor of 18? How do you know? Is 2 a factor of 18? 3? 4? 5? 6? Can you list the factors of 18 from this information? | | | | |
| Possible Student Misconceptions | Assessing and Advancing Questions | | | | |
| a) Students draw/model arrays using 18 tiles but have difficulty writing multiplication equations for each array, have one array with one equation, or have equations with no arrays. | Does the number of tiles in your array change if you turn the array a different way? Can you put the 18 tiles into equal groups? If I told you that this row is a group, how many tiles are in this group? Can you show me how you model 18 tiles into 2 rows? Can you show me how you model 18 tiles so that there are 6 tiles in each row? | | | | |
| b) Students fail to list all of the factors. | How do you know you have all possible solutions? When thinking about the 1 by 18, then the 2 by 9, what would come next? Why? Can you show an array that has 3 equal groups? (Use one of the factors that the student has not shown.) | | | | |
| Entry/Extensions | Assessing and Advancing Questions | | | | |
| If students can't get started | What is something you know about the problem? What is an array? What is a factor? Would it help to begin with repeated addition? How could you make equal groups with 18 tiles? | | | | |
| If students finish early | Can you draw an array for 4 x □ = 20? Can you write two different word problems about 18 tiles to show 3x6 and 6x3? How would your arrays change if you had 2 more tiles? | | | | |

Task 1 11

Discuss/Analyze

Whole Group Questions

Recall the EUs for this task:

- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.
- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- An array involves joining equal groups and is one way to think about multiplication and division.
- Multiplication can be used to find the total number of objects when there are a specific number of groups with the same number of objects.

Since one of the essential understandings of this task (and arc) pertains to the variety and richness of possible mathematical explanations, the whole group discussion should highlight different strategies used to solve the problem and to explain reasoning. Emphasize understanding of how the fact families relate multiplication and division.

- What are some different strategies you used to solve this problem?
- What ideas have you learned before that were helpful in solving this problem?
- How can you use arrays to model multiplication and division?
- How does the array represent two factors in a multiplication equation?
- Where is the product found in the array model?
- How does the array represent two divisors in a division equation?
- Can you draw an array with the same factors a different way?
- Which property supports 2x9=9x2? Can you show arrays illustrating this?
- How can you write a set of related multiplication and division facts?

Task 1 12

| Name | ! | | | | | |
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Task 2: Soccer Goals

Aaron is making a picture graph. Each picture of a soccer ball stands for 4 goals he scored for his team. The row for January has 6 soccer balls.

January

February









March

a) How many goals did Aaron score during January? Write a multiplication equation to show how you solved for the answer.

b) If he scores 48 goals in February, how many soccer balls should he put on the graph? How do you know?

Tennessee Department of Education: Lesson Guide 2

Task 2: Soccer Goals Aaron is making a picture graph. Each picture of a soccer ball stands for 4 goals he scored for his team. The row for January has 6 soccer balls. a) How many goals did Aaron score during January? Write a multiplication equation to show how you solved for the answer. b) If he scores 48 goals in February, how many soccer balls should he put on the graph? How do you know?

Teacher Notes:

Lessons and discussions about picture graphs are essential for teaching younger children how to organize and analyze data. It gives them a visual perspective, allowing them to quickly compare and contrast the information within the graph using pictures or symbols to represent an assigned amount of data.

| Tennessee State Standards for Mathematical | Tennessee State Standards for Mathematical |
|--|--|
| Content | Practice |
| 3.OA.A.2 Interpret whole number quotients of whole | Make sense of problems and persevere in |
| numbers, e.g., interpret 56÷8 as the number of | solving them. |
| objects in each share when 56 objects are partitioned | 2. Reason abstractly and quantitatively. |
| equally into 8 shares, or as a number of shares when | 3. Construct viable arguments and critique the |
| 56 objects are partitioned into equal shares of 8 | reasoning of others. |
| objects. | 4. Model with mathematics. |
| 3.OA.A.3 Use multiplication and division within 100 | 5. Use appropriate tools strategically. |
| to solve word problems in situations involving equal | 6. Attend to precision. |
| groups, arrays, and measurement quantities, e.g., by | 7. Look for and make use of structure. |
| using drawings and equations with a symbol for the | 8. Look for and express regularity in repeated |
| unknown number to represent the problem. | reasoning. |

Essential Understandings:

- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.
- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- Multiplication can be used to find the total number of objects when there are a specific number of groups with the same number of objects.

| Explore Phase | |
|--|--|
| Possible Solution Paths | Assessing and Advancing Questions |
| a) Aaron scored 24 goals in January. Students solve for the answer of 24 using equations: 4x6=24 or 6x4=24. | Assessing Questions: Can you explain your work to another student? How did you know what numbers to multiply? Can you explain how you thought about putting the soccer balls together? If you were to put a key on this graph, what would it say? What do the 4 and 6 represent in 4x6 and 6x4? Advancing Questions: How many goals does each soccer ball represent? How many goals are represented by two soccer balls? |

Task 2 14

| | How many soccer balls are on the graph for January? |
|---|---|
| b) February should have 12 soccer balls. Students reason that 48÷4=12, solving for a quotient, or ? x 4 =48 solving for the missing factor of 12. | Assessing Questions: Can you explain your work to another student? Can you write a multiplication/division equation to show the number of soccer balls? What are the other related math facts for this fact family? Advancing Questions: How many goals does each soccer ball represent? Can you make groups of that size out of 48? How many groups did you make? |
| Possible Student Misconceptions | Assessing and Advancing Questions |
| a) Students think each soccer ball represents 1 goal rather than 4, with a total of 6 for January, using 6 and 1 as the factors in the equation. | Can you read the second sentence and tell what each soccer ball is worth? How many goals does each soccer ball represent? How many goals are represented by two soccer balls? How many soccer balls are on the graph for January? |
| Entry/Extensions | Assessing and Advancing Questions |
| If students can't get started | What is something you know about the problem? What is the problem asking? How many goals does each soccer ball represent? How many goals are represented by two soccer balls? How many soccer balls are on the graph for January? Could you solve for an answer if each soccer ball stood for one goal? Two goals? |
| If students finish early | How can you compare the totals for January and February? How would the totals change if each soccer ball represented two goals? Six goals? Can you represent this problem with arrays? |

Task 2 15

Discuss/Analyze

Whole Group Questions

Recall the EUs for this task:

- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.
- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- Multiplication can be used to find the total number of objects when there are a specific number of groups with the same number of objects.

Since one of the essential understandings of this task (and arc) pertains to the variety and richness of possible mathematical explanations, the whole group discussion should highlight different strategies used to solve the problem and to explain reasoning. Emphasize understanding of how the fact families relate multiplication and division.

- What are some different strategies you used to solve this problem?
- What ideas have you learned before that were helpful in solving this problem?
- If Aaron scores 36 goals in March, can you complete the graph including a key with soccer balls?
- How can you use multiplication to solve division problems?
- How can you use division to solve multiplication problems?

Task 2 16

| Name | | | | | |
|------|------|------|------|------|------|
| | | | | | |

Task 3: Name That Number

1. Mia bought 2 copies of the same book. She spent \$18. What was the cost of one book? Write an equation and explain your reasoning.



2. Mr. Martin buys 36 biscuits for a class breakfast. He places them on plates for his students. If he places 12 biscuits on each plate, how many plates does he need? Write an equation and explain your reasoning.

Tennessee Department of Education: Lesson Guide 3

Task 3: Name That Number

3rd Grade

- 1. Mia bought 2 copies of the same book. She spent \$18. What was the cost of one book? Write an equation and explain your reasoning.
- 2. Mr. Martin buys 36 biscuits for a class breakfast. He places them on plates for his students. If he places 12 biscuits on each plate, how many plates does he need? Write an equation and explain your reasoning.

Teacher Notes:

While most students will naturally divide to solve both of these problems, teachers should emphasize the relationship between multiplication and division. Note that the explanation of reasoning should require students to use words and possibly drawings to explain their thoughts.

| Tennessee State Standards for Mathematical | Tennessee State Standards for Mathematical |
|--|--|
| Content | Practice |
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| objects in each share when 56 objects are partitioned | 2. Reason abstractly and quantitatively. |
| equally into 8 shares, or as a number of shares when | 3. Construct viable arguments and critique the |
| 56 objects are partitioned into equal shares of 8 | reasoning of others. |
| objects. | 4. Model with mathematics. |
| 3.OA.A.3 Use multiplication and division within 100 | 5. Use appropriate tools strategically. |
| to solve word problems in situations involving equal | 6. Attend to precision. |
| groups, arrays, and measurement quantities, e.g., by | 7. Look for and make use of structure. |
| using drawings and equations with a symbol for the | 8. Look for and express regularity in repeated |
| unknown number to represent the problem. | reasoning. |

Essential Understandings:

- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.
- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- An array involves joining equal groups and is one way to think about multiplication and division.
- Multiplication can be used to find the total number of objects when there are a specific number of groups with the same number of objects.

| groups with the same number of objects. | | | | |
|---|---|--|--|--|
| Explore Phase | | | | |
| Possible Solution Paths | Assessing and Advancing Questions | | | |
| 1. The books each cost \$9. | Assessing Questions:Why did you choose to write this equation? | | | |
| Students may use any of the following equations: $\$18 \div 2 = \9 , $\$18 \div \square = 2$, $2 \times \square = \$18$, or | Can you explain how you used your numbers in the equation? Can you use a different strategy to show how you can solve for the answer? | | | |
| x 2 = \$18. For any of the equations with the unknown part, students should indicate that part is \$9. | Advancing Questions: What is the total amount spent on the books? How many books did Mia buy? Can you use arrays to help solve the problem? | | | |
| Students should accompany their equations with a written description of their reasoning. Answers will vary. | Can you tell me about the groups and how much is in each group? Could you use a symbol in the equation for the unknown? | | | |

| 2. Each plate will hold 12 biscuits. Students may use any of the following equations: 36 ÷ 3 = 12, 36 ÷ □ = 3, 3 x □ = 36, or □ x 3 = 36. For any of the equations with the unknown part, students should indicate that part is 12 biscuits. Students should accompany their equations with a written description of their reasoning. Answers will vary. | Assessing Questions: Why did you choose to write this equation? Can you explain how you used your numbers in the equation? Can you use a different strategy to show how you can solve for the answer? Advancing Questions: What is the total number of biscuits? How many biscuits does Mr. Martin place on each plate? Can you use arrays to help solve the problem? Can you tell me about the groups and how much is in each group? |
|---|---|
| Possible Student Misconceptions | Assessing and Advancing Questions |
| Students use the 2 and 18 as factors and solve for the product of 36. | What is the total amount spent on the books? How many books did Mia buy? Can you use a drawing to help you solve this problem? Can you use arrays to help solve the problem? What number could you write on each of the two books? Are you breaking apart 18 or putting it together? How do you know? What operation is this? Can you write the equation to represent what you just said? |
| Entry/Extensions | Assessing and Advancing Questions |
| If students can't get started | What is something you know about the problem? What is the total amount spent on the books? How many books did Mia buy? Can you draw a diagram? If two books cost \$10 each, how much money would Mia need to buy them? Can that help you reason about Problem 1? |
| If students finish early | How much money does Mia need to buy 3 books at the same price? What happens to the total each time you add a book? If Mr. Martin had 4 plates and 36 biscuits, how many biscuits would go on each plate? Can you write this information in an equation? |

Discuss/Analyze

Whole Group Questions

Recall the EUs for this task:

- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.
- An array involves joining equal groups and is one way to think about multiplication.
- Multiplication can be used to find the total number of objects when there are a specific number of groups with the same number of objects.

Since one of the essential understandings of this task (and arc) pertains to the variety and richness of possible mathematical explanations, the whole group discussion should highlight different strategies used to solve the problem and to explain reasoning. Emphasize understanding of how the fact families relate multiplication and division.

- What are some different strategies you used to solve this problem?
- What does it mean to have related math facts using multiplication and division?
- Did anyone use arrays to solve these problems? What would arrays look like using the information in problems 1 and 2?
- If Mr. Martin had 4 plates and 36 biscuits, how many biscuits would go on each plate?

| Name | | | | | | |
|------|--|--|--|--|--|--|
| | | | | | | |

Task 4: Surprise Birthday Party

Miranda's parents bought 24 balloons for Miranda and some friends to share at her birthday party.



a) If she and 5 friends share the balloons, how many will each child receive? Write an equation and explain your reasoning.

b) Miranda decides instead to make bundles of 3 balloons to tie around the room. How many bundles would she be able to make?

Tennessee Department of Education: Lesson Guide 4

Task 4: Surprise Birthday Party

3rd Grade

Miranda's parents bought 24 balloons for Miranda and some friends to share at her birthday party.

- a) If she and 5 friends share the balloons, how many will each child receive? Write an equation and explain your reasoning.
- b) Miranda decides instead to make bundles of 3 balloons to tie around the room. How many bundles would she be able to make?

Teacher Notes:

Note that the explanation of reasoning should require students to use words and a possibly drawings to explain their thoughts. Emphasize the notion of an unknown number to target standards 3.OA.A.4 and 3.OA.B.6.

| Tennessee State Standards for Mathematical | Tennessee State Standards for Mathematical |
|--|--|
| Content | Practice |
| 3.0A.A.2 Interpret whole number quotients of whole | |
| numbers, e.g., interpret 56÷8 as the number of | |
| objects in each share when 56 objects are partitioned | |
| equally into 8 shares, or as a number of shares when | |
| 56 objects are partitioned into equal shares of 8 | 1. Make sense of problems and persevere in |
| objects. | solving them. |
| 3.OA.A.3 Use multiplication and division within 100 | 2. Reason abstractly and quantitatively. |
| to solve word problems in situations involving equal | 3. Construct viable arguments and critique the |
| groups, arrays, and measurement quantities, e.g., by | reasoning of others. |
| using drawings and equations with a symbol for the | 4. Model with mathematics. |
| unknown number to represent the problem. | 5. Use appropriate tools strategically. |
| 3.OA.A.4 Determine the unknown whole number in a | 6. Attend to precision. |
| multiplication or division equation relating three | 7. Look for and make use of structure. |
| whole numbers. For example, determine the unknown | 8. Look for and express regularity in repeated |
| number that makes the equation true in each of the | reasoning. |
| equations $8 \times ? = 48, 5 = _ \div 3, 6 \times 6 = ?$ | |
| 3.OA.B.6 Understand division as an unknown-factor | |
| problem. For example, find 32 ÷ 8 by finding the | |
| number that makes 32 when multiplied by 8. | |

Essential Understandings:

- Division can be used to find how many equal groups (measurement-repeated subtraction) or how many are in each group (partitive-sharing).
- Multiplication and division have an inverse relationship and can be used to find division or multiplication facts.
- Any division problem can be thought of as a multiplication fact with a missing factor.
- Mathematical explanations can be given using words, objects, pictures, numbers or equations.

| Explore Phase | | | | |
|--|---|--|--|--|
| Possible Solution Paths | Assessing and Advancing Questions | | | |
| a) Each child will receive 4 balloons. Equations may | Assessing Questions: | | | |
| include $24 \div 6 = 4$, $6x = 24$ or others from the related | Can you explain your work to another student? | | | |
| math facts with knowledge that 4 is the unknown. | How does your equation and/or drawing prove | | | |
| Explanations will vary. An example: | you know how many balloons each gets? | | | |
| | What word(s) helped you to know to use this | | | |
| There should be 6 groups of balloons. In order | strategy? | | | |

for all 6 to receive an equal number of balloons, each group would contain 4 balloons because 6x4=24.

Some students may use repeated addition or subtraction in their drawing and equations.

Guide the students back to the standard using multiplication or division equations such as:

$$24 \div 6 = 4$$
, $24 \div 4 = 6$, $6x = 4 = 24$, or $4x = 6 = 24$

b) There are enough balloons for 8 bundles. Equations may include $24 \div 3 = 8$, 3x = 24, or others from the related math facts with knowledge that 8 is the unknown. Explanations will vary. An example:

There should be 8 bundles of balloons. In order create bundles of 3, there should be 8 bundles because $24 \div 3 = 8$.

Some students may use repeated addition or subtraction in their drawing and equations.

Guide the students back to the standard using multiplication or division equations such as:

- Can you use a different strategy to show your answer?
- Would your answer change if the balloons were shared between Miranda and 3 friends?
- Did you use a symbol to represent the unknown value?

Advancing Questions:

- How many groups are there?
- How many balloons are there?
- Can you use a drawing to show how many balloons should be in each group?
- What operation can you use to show how to share among equal groups?
- Can you show this using multiplication or division?
- Can you use a symbol to represent the unknown value?

Assessing Questions:

- Can you explain your work to another student?
- How does your equation and/or drawing prove you know how many bundles there should be?
- What word(s) helped you to know to use this strategy?
- Can you use a different strategy to show your answer?
- Would your answer change if the balloons were tied in bundles of 8?
- Did you use a symbol to represent the unknown value?

Advancing Questions:

- How many balloons are in each bundle?
- How many total balloons are there?
- Can you use a drawing to show how many bundles there should be?
- What operation can you use to show how to share among equal groups?
- Can you show this using multiplication or division?
- Can you use a symbol to represent the unknown value?

Possible Student Misconceptions

a) Students share the balloons among the 5 friends, leaving out Miranda. The drawings and/or equations show that each friend receives 4 balloons for a total of 20, leaving the remaining 4 out of the total.

Assessing and Advancing Questions

- How many children are at the party?
- Will Miranda get some balloons?
- How does that change your answer?

| Entry/Extensions | Assessing and Advancing Questions | | |
|-------------------------------|---|--|--|
| If students can't get started | What is something you know about the problem? If you reread the first sentence of the problem, who will get balloons? Would it help to have counters or a drawing to model the balloons you are sharing? How many counters would you need to model part a? Part b? | | |
| If students finish early | Can you solve parts a) and b) if there were 36 balloons? | | |

Discuss/Analyze

Whole Group Questions

Recall the EUs for this task:

- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- Multiplication and division have an inverse relationship and can be used to find division or multiplication facts.
- Any division problem can be thought of as a multiplication fact with a missing factor.
- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.

Since one of the essential understandings of this task (and arc) pertains to the variety and richness of possible mathematical explanations, the whole group discussion should highlight different strategies used to solve the problem and to explain reasoning. Emphasize division as an unknown factor problem.

- What are some different strategies you used to solve this problem?
- What is the relationship between multiplication and division?
- How is it that we can solve for the answer with only one factor and the product?
- What operation can you use to show how to share among equal groups?
- What are some clue words that help to determine if you should think about multiplication or division?

Task 5: Pin the Number on the Chart

Miranda's mother needs to get some supplies for the surprise birthday party for her daughter. Help her complete the supply list based on the information below.

| Item | Cost per Pack | Number of packs purchased | Total Cost |
|---------|---------------|---------------------------|------------|
| Bowls | \$4 | | |
| Cups | \$3 | | \$9 |
| Napkins | | 2 | \$6 |

a) What is the cost of each pack of napkins if she spent \$6 buying 2 packs of napkins? Solve by writing an equation with an unknown and fill in the chart.

b) If each pack of cups cost \$3, how many packs can she buy with \$9? Solve by writing an equation with an unknown and fill in the chart.

c) There are 6 bowls in one pack and 18 napkins in one pack. If Miranda's mother has 2 packs of napkins, how many packs of bowls will she need to buy if she wants to have an equal amount of bowls and napkins? Solve by writing an equation with an unknown and fill in the chart.

d) What will be the total cost of bowls? Solve by writing an equation with an unknown and fill in the chart.

Tennessee Department of Education: Lesson Guide 5

Task 5: Pin the Number on the Chart

3rd Grade

Miranda's mother needs to get some supplies for the surprise birthday party for her daughter. Help her complete the supply list based on the information below.

a) What is the cost of each pack of napkins if she spent \$6 buying 2 packs of napkins? Solve by writing an equation with an unknown and fill in the chart.

number that makes 32 when multiplied by 8.

| Item | Cost per Pack | Number of packs purchased | Total Cost |
|---------|---------------|---------------------------|------------|
| Bowls | \$4 | | |
| Cups | \$3 | | \$9 |
| Napkins | | 2 | \$6 |

- b) If each pack of cups cost \$3, how many packs can she buy with \$9? Solve by writing an equation with an unknown and fill in the chart.
- c) There are 6 bowls in one pack and 18 napkins in one pack. If Miranda's mother has 2 packs of napkins, how many packs of bowls will she need to buy if she wants to have an equal amount of bowls and napkins? Solve by writing an equation with an unknown and fill in the chart.
- d) What will be the total cost of bowls? Solve by writing an equation with an unknown and fill in the chart.

Teacher Notes:

Students may use repeated addition to solve for the solution totals. Teacher should guide students' thinking towards the standard by using multiplication and/or division equations with the unknowns identified in each equation. Encourage the use of a symbol for the unknown.

| Tennessee State Standards for Mathematical | Tennessee State Standards for Mathematical |
|--|--|
| Content | Practice |
| 3.OA.A.2 Interpret whole number quotients of whole | |
| numbers, e.g., interpret 56÷8 as the number of | |
| objects in each share when 56 objects are partitioned | |
| equally into 8 shares, or as a number of shares when | |
| 56 objects are partitioned into equal shares of 8 | 1. Make sense of problems and persevere in |
| objects. | solving them. |
| 3.OA.A.3 Use multiplication and division within 100 | 2. Reason abstractly and quantitatively. |
| to solve word problems in situations involving equal | 3. Construct viable arguments and critique the |
| groups, arrays, and measurement quantities, e.g., by | reasoning of others. |
| using drawings and equations with a symbol for the | 4. Model with mathematics. |
| unknown number to represent the problem. | 5. Use appropriate tools strategically. |
| 3.0A.A.4 Determine the unknown whole number in a | 6. Attend to precision. |
| multiplication or division equation relating three | 7. Look for and make use of structure. |
| whole numbers. For example, determine the unknown | 8. Look for and express regularity in repeated |
| number that makes the equation true in each of the | reasoning. |
| equations $8 \times ? = 48, 5 = _ \div 3, 6 \times 6 = ?$ | |
| 3.OA.B.6 Understand division as an unknown-factor | |
| problem. For example, find 32 ÷ 8 by finding the | |

Essential Understandings:

- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- Multiplication and division have an inverse relationship and can be used to find division or multiplication facts
- Any division problem can be thought of as a multiplication fact with a missing factor.
- Any multiplication problem can be thought of as a division problem with a missing dividend or divisor.
- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.

| Evol | lara | Phase |
|------|-------|--------|
| EXP | ioi e | riiase |

a) Students write one of the following equations:

Possible Solution Paths

In order to spend \$6, each pack must cost \$3 since 3 is the unknown in all of the above equations.

Student should place a 3 in the appropriate cell of the table on the bottom row.

b) Students write one of the following equations:

Miranda's mother will need to purchase 3 packs of cups to spend \$9 total since 3 is the unknown in all of the above equations.

Student should place a 3 in the appropriate cell on the third row.

c) Students show that the total number of napkins is 36 since 2x18=36.

Equations to show the number of bowls can be

since bowls come in packs of 6. The unknown is 6, so she should purchase 6 packs of bowls. Students should fill in the appropriate cell of the table.

Assessing and Advancing Questions

Assessing Questions:

- What do the numbers represent in your equation?
- Did you think about putting the packs together or breaking up the \$4?
- Is there a way to check your answer?

Advancing Questions:

- What is the unknown in this problem? How can you represent an unknown in an equation?
- How much money did she spend on napkins?
 How many packs of napkins did she buy with \$6?
- Can you draw a picture to help you solve this?
- What operations relate to equal groups/sharing?

Assessing Questions:

- What do the numbers represent in your equation? What did you use to represent the unknown?
- What helped you to decide to use this strategy?
- Is there a way to check your answer?

Advancing Questions:

- What is the unknown in this problem? How can you represent an unknown in an equation?
- How much money is she spending on cups?
- How does the cost per pack relate to the total amount?
- Can you draw a picture to help you solve this?
- What operations relate to equal groups/sharing?

Assessing Questions:

- What do the numbers represent in your equation? What did you use to represent the unknown?
- Can you explain your strategy?

Advancing Questions:

- What is the unknown in this problem? How can you represent an unknown in an equation?
- What information do you need to determine

| buy 6 packs \$4x6=0 The total co should finis | ost for bowls sho sh the last cell of | rite the equatould be \$24. State table. | tion | how many packs bowls Miranda's mother needs? How can you find out how many napkins she has? How many bowls do you need? How many bowls are in a pack? Can you draw a picture to help you solve this? What operations relate to equal groups/sharing? Assessing Questions: Why did you choose to use this equation? Is there a way to check your answer? Advancing Questions: How many packs of bowls had to be purchased? What is the cost per pack? Can you draw a picture to help you solve this? |
|--|---|--|------|--|
| | udent Misconce | - | | Assessing and Advancing Questions |
| cups and na addition as | a-b) Students use the known parts of the chart (for cups and napkins) to solve for unknowns using addition as if each blank was a missing addend, as in the chart below. | | | Can you explain what you notice about the chart? How much money did she spend on napkins? How many packs of napkins did she buy with \$6? |
| | | purchased | | Can you draw a picture to help you solve this? |
| Bowls | \$4 | | | What operations relate to equal groups/sharing? |
| Cups | \$3 | 3 | \$9 | Does your answer make sense?How many groups of \$3 can be made to show a |
| Napkins | \$3 | 2 | \$6 | total of \$9? |
| Entry/Exte | nsions | | | Assessing and Advancing Questions |
| If students can't get started | | | | Can you explain what you notice about the chart? What operations relate to equal groups/sharing? Can you draw pictures to represent the information from the chart? Can you show how many groups of 2 it takes to make 8? Etc. Would it help if you covered up the rows of the chart you are not currently working on? |
| If students finish early | | | | How can you find the amount of money needed to purchase 6 packs of cups? Can you write about what you have learned or found out today? Can you find another strategy to work these problems? |

Discuss/Analyze

Whole Group Questions

Recall the EUs for this task:

- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- Multiplication and division have an inverse relationship and can be used to find division or multiplication facts.
- Any division problem can be thought of as a multiplication fact with a missing factor.
- Any multiplication problem can be thought of as a division problem with a missing dividend or divisor.
- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.

Since one of the essential understandings of this task (and arc) pertains to the variety and richness of possible mathematical explanations, the whole group discussion should highlight different strategies used to solve the problem and to explain reasoning. Emphasize writing equations with an unknown.

- What are some different strategies you used to solve this problem?
- What ideas have you learned before that were helpful in solving this problem?
- Can multiplication and division be used to solve for each item from above with just the information from the chart? Why or why not?
- How can you check your answers? What is the relationship between multiplication and division?
- Do the pictures and equations represent the same thing? If so, how?

| Name |
|------|
| Name |

Task 6: Musical Chairs

Miranda has 5 tables.

a) If 30 people will be attending her party, how many chairs should be at each table to form equal groups? Solve by writing an equation with an unknown.



b) If Miranda has 45 flowers and she wants to put a vase with the same number of flowers on each table, how many flowers should be in each vase? Solve by writing an equation with an unknown.

Task 6: Musical Chairs 3rd Grade

Miranda has 5 tables.

- a) If 30 people will be attending her party, how many chairs should be at each table to form equal groups? Solve by writing an equation with an unknown.
- b) If Miranda has 45 flowers and she wants to put a vase with the same number of flowers on each table, how many flowers should be in each vase? Solve by writing an equation with an unknown.

Teacher Notes:

Emphasize the notion of an unknown number to target standards 3.OA.A.4 and 3.OA.B.6.

| Emphasize the notion of an unknown number to target standards 5.0A.A.4 and 5.0A.B.0. | | |
|--|--|--|
| Tennessee State Standards for Mathematical | Tennessee State Standards for Mathematical | |
| Content | Practice | |
| 3.OA.A.2 Interpret whole number quotients of whole | | |
| numbers, e.g., interpret 56÷8 as the number of | | |
| objects in each share when 56 objects are partitioned | | |
| equally into 8 shares, or as a number of shares when | | |
| 56 objects are partitioned into equal shares of 8 | 1. Make sense of problems and persevere in | |
| objects. | solving them. | |
| 3.OA.A.3 Use multiplication and division within 100 | 2. Reason abstractly and quantitatively. | |
| to solve word problems in situations involving equal | 3. Construct viable arguments and critique the | |
| groups, arrays, and measurement quantities, e.g., by | reasoning of others. | |
| using drawings and equations with a symbol for the | 4. Model with mathematics. | |
| unknown number to represent the problem. | 5. Use appropriate tools strategically. | |
| 3.OA.A.4 Determine the unknown whole number in a | 6. Attend to precision. | |
| multiplication or division equation relating three | 7. Look for and make use of structure. | |
| whole numbers. For example, determine the unknown | 8. Look for and express regularity in repeated | |
| number that makes the equation true in each of the | reasoning. | |
| equations $8 \times ? = 48, 5 = _ \div 3, 6 \times 6 = ?$ | | |
| 3.OA.B.6 Understand division as an unknown-factor | | |
| problem. For example, find 32 ÷ 8 by finding the | | |

Essential Understandings:

number that makes 32 when multiplied by 8.

- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- Multiplication and division have an inverse relationship and can be used to find division or multiplication facts.
- Any division problem can be thought of as a multiplication fact with a missing factor.
- Any multiplication problem can be thought of as a division problem with a missing dividend or divisor.
- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.

| Explore Phase | |
|---|--|
| Possible Solution Paths | Assessing and Advancing Questions |
| a) Students write one of the following equations: | Assessing Questions: |
| 30 ÷ □= 5 | What do the numbers represent in your |
| 5 x □= 30 | equations? |
| 30 ÷ 5= □ | What does the unknown value represent? |
| In order to form equal groups, there should be 6 | How did you know to multiply/divide? |
| chairs at each table since 6 is the unknown in all of | Could you solve this problem using a different |
| the above equations. | operation? |

| | Advancing Questions: Could you draw a picture to model this problem? What operation can you use to show how to share among equal groups? What is the unknown in this problem? Can you write an equation that includes a placeholder for the unknown? |
|---|---|
| b) Students write one of the following equations: 45 ÷ □= 5 5 x □= 45 45 ÷ 5= □ In order to form equal groups, there should be 9 flowers in each vase since 9 is the unknown in all of the above equations. | Assessing Questions: Can you explain your work to a friend? What do the numbers represent in your equations? What does the unknown value represent? How did you know to multiply/divide? Could you solve this problem using a different operation? Advancing Questions: Could you draw a picture to model this problem? What operation can you use to show how to share among equal groups? What is the unknown in this problem? Can you write an equation that includes a placeholder for the unknown? |
| Possible Student Misconceptions | Assessing and Advancing Questions |
| Students may not realize that the unknown can be anywhere in the equation. Thus, they only understand that they can write $30 \div 5 = \Box$ or $45 \div 5 = \Box$. | What is the unknown in this problem? What multiplication fact family relates the values in this problem? How can you represent an unknown in an equation? |
| Entry/Extensions | Assessing and Advancing Questions |
| If students can't get started | What is something you know about the problem? What is the unknown in this problem? Can you use counters or draw a picture to help you solve this problem? How can you represent an unknown in an equation? What operation can you use to show how to share among equal groups? |
| If students finish early | If thirty people are attending Miranda's party, what are all the possible combinations of tables and chairs if there are an equal number of chairs at each table? |

Discuss/Analyze

Whole Group Questions

Recall the EUs for this task:

- Division can be used to find how many equal groups (measurement/repeated subtraction) or how many are in each group (partitive/sharing).
- Multiplication and division have an inverse relationship and can be used to find division or multiplication facts.
- Any division problem can be thought of as a multiplication fact with a missing factor.
- Any multiplication problem can be thought of as a division problem with a missing dividend or divisor.
- Mathematical explanations can be given using words, objects, pictures, numbers, or equations.

Since one of the essential understandings of this task (and arc) pertains to the variety and richness of possible mathematical explanations, the whole group discussion should highlight different strategies used to solve the problem and to explain reasoning. Emphasize writing equations with an unknown.

- What are some different strategies you used to solve this problem?
- What ideas have you learned before that were helpful in solving this problem?
- How are multiplication and division representations related to one another?
- How do you know when to use multiplication to solve a problem?
- How do you know when to use division to solve a problem?
- How can you represent an unknown in a multiplication or division equation?