**Tennessee Comprehensive Assessment Program / Mathematics** 

# TCAP/CRA 2012-2013



## Task 2: Three Problems Task Scoring Guide

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## Task 2. Three Problems Task

Problem 1	Problem 2	Problem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?

a. Solve Problem 1, Problem 2, and Problem 3. Draw a picture and write an equation that shows how you determined the solution to each problem.

Problem 1	Problem 2	Problem 3
	Page 8 GO	ON TO THE NEXT PAGE.

b. Frank sees that the math in the problems is similar because the problems all include
32 feet of ribbon. Explain another way the math in the problems is similar, or a way the math in the problems is different. (Do not state that the problems contain the same numbers.)

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#### 2. Three Problems Task Scoring Guide

#### The CCSS for Mathematical Content (3 points)

- 3.OA.2(a) The student writes at least two multiplication or division equations or expressions that indicate awareness that 32 can be partitioned in one of the following ways: 4 iterations of some length, 8 iterations of some length, or lengths that are 8 or 4 feet long. (The response does not need to be correct in order to determine that the student interprets the relationship between 32 and 8 or 4.)
- 3.OA.2(b) The student shows a drawing or a description that indicates an understanding of division as an equal partitioning of a whole. Work shows or describes iterations of lengths of 8 or 4 or 4 cuts that are 8 feet or 4 feet, depending on the context.
- 3.OA.3 The student uses multiplication or division to correctly solve at least two of the given problems.

Total Content Points \_\_\_\_\_

#### The CCSS for Mathematical Practice (5 points)

MP1 The student makes sense of division by determining the unknown quotient and comparing the solution paths for the three problems. Gives diagrams and equations for each problem, and a comparison of the situations is made.

(MP1: Make sense of problems and persevere in solving them.)

MP2 The student writes equations and re-contextualizes the equations within the context of the problems. Provides labels, pictures, or diagrams with the length indicated in problem 1 and the number of equal lengths indicated in problems 2 and 3, thereby contextualizing the mathematical equations with the situation.

(MP2: Reason abstractly and quantitatively.)

MP4 The student creates diagrams and/or equations that indicate an understanding of how to model division situations.

(MP4: Model with mathematics.)

MP6 The student provides equations and diagrams with labels that accurately represent the context.

(MP6: Attend to precision.)

- MP7 The student indicates an understanding of the structure of the situations. The student may do this in one of the following ways:
  - by indicating that Problems 2 and 3 both provide the length of the pieces.
  - by indicating that Problems 2 and 3 both ask students to identify the number of cuts possible.
  - by indicating that Problems 1 and 2 differ from each other because Problem 2 indicates the length of the cut and asks about the number of cuts, whereas Problem 1 asks about the length of the cuts and provides information about the number of cuts.
  - by indicating that division or multiplication is a possible solution path for all three problems.

(MP7: Look for and make use of structure.)

Total Practice Points \_\_\_\_\_

Total Awarded Points \_\_\_\_\_

## The CCSS for Mathematical Content Addressed in This Task

#### Represent and solve problems involving multiplication and division.

- 3.OA.2 Interpret whole-number quotients of whole numbers; e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as  $56 \div 8$ .
- 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

#### The CCSS for Mathematical Practice\*

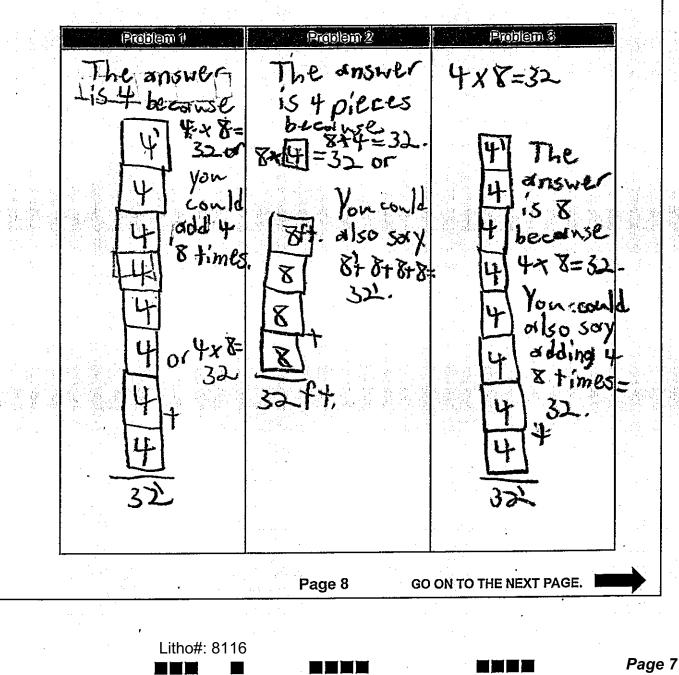
- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

\*Gray text indicates Mathematical Practices that are not addressed in this task.

Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

#### Task 2. Three Problems Task

Problem 1	Problem 2	Problem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?



## Guide 1b

b. Frank sees that the math in the problems is similar because the problems all include
 32 feet of ribbon. Explain another way the math in the problems is similar, or a way the math in the problems is different. (Do not state that the problems contain the same numbers.)

The most h problems are similar because they all have to do with the source instiplication problem. 14x & and 8+4.



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Guide 1	Litho 8116
Total Content Points: 3	(3.OA.2(a), 3.OA.2(b), 3.OA.3)
Total Practice Points: 5	(MP1, MP2, MP4, MP6, MP7)

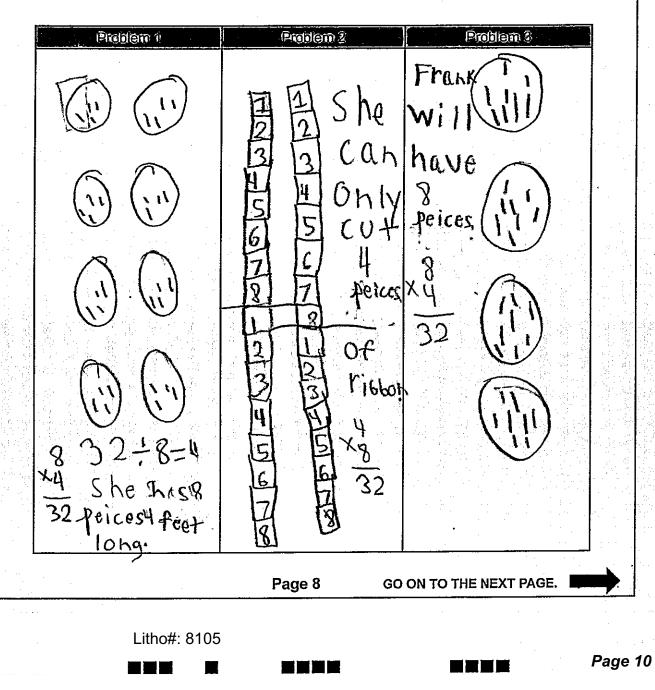
This student correctly creates multiplication equations to determine how the ribbons should be partitioned (3.OA.2(a)), and provides clear and accurate diagrams indicating the partitioning of the ribbons into lengths of 8 or 4 feet, respectively (3.OA.2(b)). The student uses multiplication,  $4 \times 8 = 32$ , to correctly solve the problems (3.OA.3). This student determines the unknown amount for each of the three problems with equations and diagrams, and makes a valid comparison of the three problems in Part B (MP1, MP4). The student contextualizes the equations and diagrams by labeling the length of the cuts in feet in the diagrams and indicating the number of pieces the ribbons will be divided into (MP2). The answers given are designated as either the number of pieces cut or the lengths pieces are cut into (MP6). The explanation and repeated addition that is shown in Part B indicates the student has an understanding of the structure of the problems (MP7).

Total Awarded Points: 8 out of 8

## Guide 2a

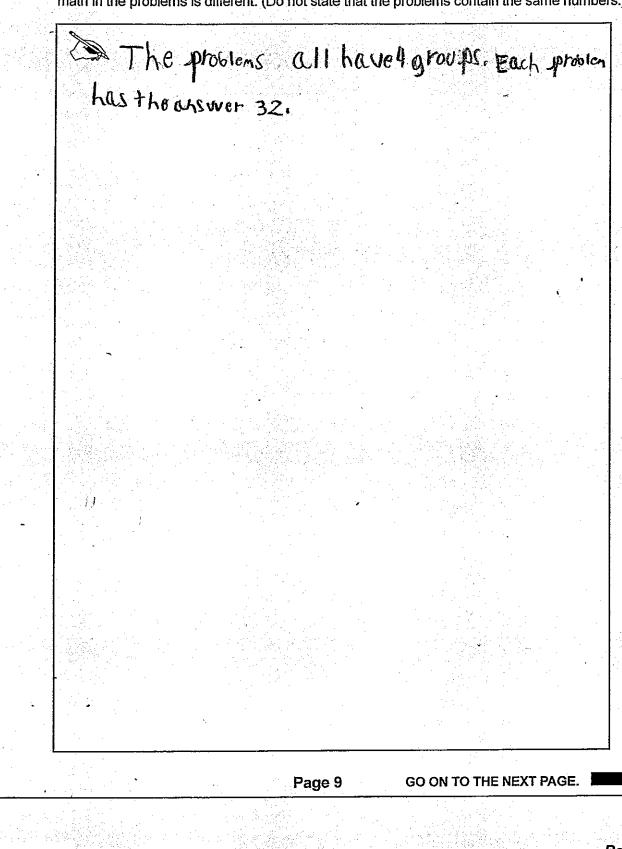
## Task 2. Three Problems Task

Problem 1	Problem 2	Piedlem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?



## Guide 2b

b. Frank sees that the math in the problems is similar because the problems all include
 32 feet of ribbon. Explain another way the math in the problems is similar, or a way the
 math in the problems is different. (Do not state that the problems contain the same numbers.)



Guide 2	Litho 8105
Total Content Points: 3	(3.OA.2(a), 3.OA.2(b), 3.OA.3)
Total Practice Points: 3	(MP2, MP4, MP6)

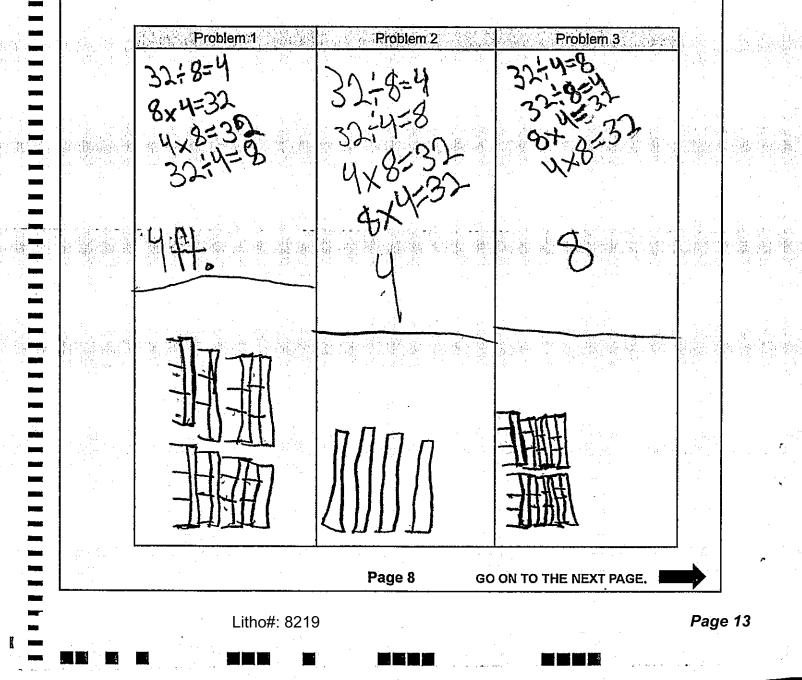
This student uses multiplication expressions and a division equation to determine the correct answers to the given problems (3.OA.2(a), 3.OA.3). The student creates diagrams that demonstrate an understanding of the given problems as division problems to describe how the ribbons were partitioned (3.OA.2(b), MP4). In Part A of the task, the diagrams and equations demonstrate the student's ability to recognize how to solve the given problems, and the correct labeling of the answers shows attention to precision and the student's ability to re-contextualize the answers once they are found (MP2, MP6). Although the work in Part A of the response shows that the student can correctly solve the given problems, the comparison given in Part B is insufficient as it does not move beyond indicating that the problems contain the same numbers and does not demonstrate a full understanding of the structure of the problems (no credit for MP7). The lack of an acceptable answer in Part B indicates that the student has not fully made sense of the problem (no credit for MP1).

Total Awarded Points: 6 out of 8

## Guide 3a

#### Task 2. Three Problems Task

Problem 1	Problem 2	Problem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?



b. Frank sees that the math in the problems is similar because the problems all include
 32 feet of ribbon. Explain another way the math in the problems is similar, or a way the math in the problems is different. (Do not state that the problems contain the same numbers.)

Problem 1 and 3. Laccording to my pictures) have the same pictures. Both problem Land 2 have to do 32+8 to find the answer

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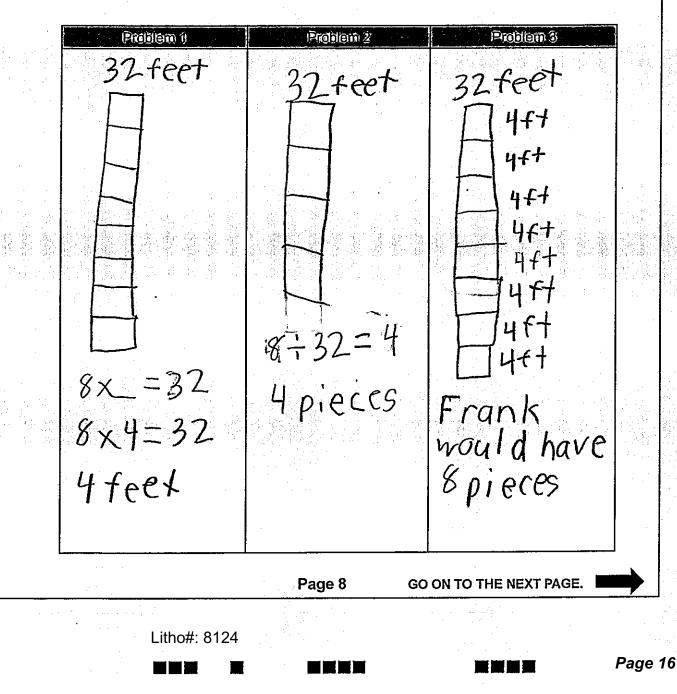
Guide 3	Litho 8219
Total Content Points: 2	(3.OA.2(a), 3.OA.3)
Total Practice Points: 3	(MP2, MP4, MP7)

This response contains equations, presented as fact families, which indicate the student's awareness of how the 32 feet of ribbon would be divided in each of the given problems (3.OA.2(a)). The multiplication (8 x 4 = 32) and division ( $32 \div 8 = 4$ ) equations are correctly solved, and the student provides the correct answer for each problem (3.OA.3). In Problem 1, the answer is labeled as the length of the partition in "ft.," making it unnecessary to label answers in Problems 2 and 3 as pieces since the prompt asks for the number of pieces in those two problems (MP2). The student sets up each problem correctly with equations and in Part B states acceptable comparisons of the problems (MP4, MP7). Although the student attempts to use diagrams to model the division problems, the diagrams used do not provide any indication of the lengths of the cut pieces (no credit for 3.OA.3(b)). The lack of labeling of the diagrams indicates a lack of attention to precision in the response (no credit for MP6), and the weakness of the diagrams means that the student has not completely responded to all parts of the task (no credit for MP1).

Total Awarded Points: 5 out of 8

## Task 2. Three Problems Task

Problem 1	Problem 2	Prodem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?



## Guide 4b

Frank sees that the math in the problems is similar because the problems all include 32 feet of ribbon. Explain another way the math in the problems is similar, or a way the math in the problems is different. (Do not state that the problems contain the same numbers.)

b.

## They both use ribon. Some are division proplems, and some are multiply cation problems.

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Guide 4	Litho 8124
Total Content Points: 3	(3.OA.2(a), 3.OA.2(b), 3.OA.3)
Total Practice Points: 2	(MP2, MP4)

This response shows only one correct equation (for Problem 1) but does provide all 3 correct answers for Part A (3.OA.3). Although the equation provided for the second problem is incorrect and there is no equation provided for the third problem, the equations shown in problems 1 and 2 indicate that the student recognizes that division and multiplication are possible ways to indicate the partitioning of the ribbon and solve the problems (3.OA.2(a)). The answers given are supported by the diagrams provided, and the third diagram indicates both the number of pieces and the length of each piece (3.OA.2(b), MP4). The student labels the answers as "feet" in Problem 1 and "pieces" in Problems 2 and 3 (MP2). The comparison made in Part B incorrectly implies that multiplication is necessary for some of the problems and division for others (no credit for MP7). Although the student recognizes that division can be used to solve the problems, and correctly labels the answers given, the incorrect division equation given for Problem 2 indicates a lack of attention to precision (no credit for MP6), and the insufficient answer to Part B shows a lack of making full sense of the task (no credit for MP1).

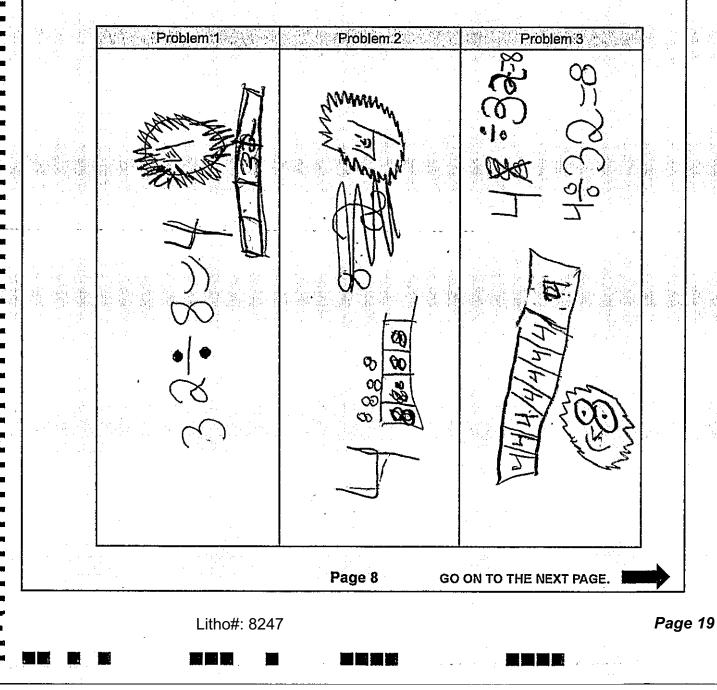
Total Awarded Points: 5 out of 8

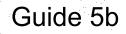
## Guide 5a

#### Task 2. Three Problems Task

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Problem 1	Problem 2	Problem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?





b. Frank sees that the math in the problems is similar because the problems all include
 32 feet of ribbon. Explain another way the math in the problems is similar, or a way the
 math in the problems is different. (Do not state that the problems contain the same numbers.)

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Guide 5	Litho 8247
Total Content Points: 3	(3.OA.2(a), 3.OA.2(b), 3.OA.3)
Total Practice Points: 2	(MP4, MP7)

This response contains three correct answers for Part A (3.OA.3). The student gives a correct equation for Problem 1 and an appropriate but incorrect equation for Problem 3, indicating that the student recognizes how to solve the problem using division (3.OA.2(a)). Although the diagram used in the first problem is insufficient (as no lengths of the individual parts are given), the diagrams in Problems 2 and 3 show correct modeling of the problems by indicating the length of the pieces cut (3.OA.2(b), MP4). The student demonstrates correct reasoning that shows an understanding of the structure of the situations by saying in Part B that the problems are all dividing, since the prompt never mentions division (MP7). The true but general statement provided tells us why all three problems are similar. The student does not label any answers or the lengths of the pieces in the drawings provided (no credit for MP2, no credit for MP6). The student does not complete the task, as no equation is given for Problem 2 (no credit for MP1).

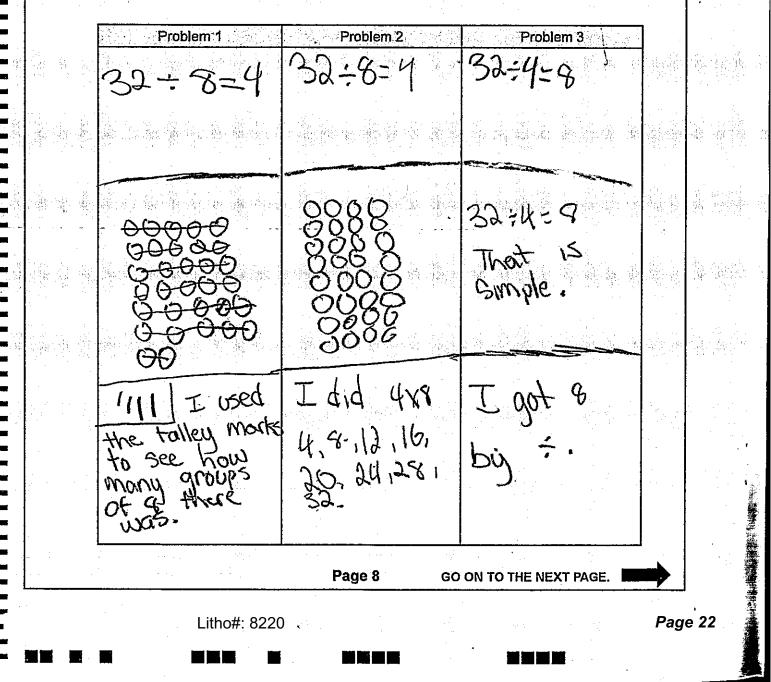
Total Awarded Points: 5 out of 8

## Guide 6a

## Task 2. Three Problems Task

Problem 1	Problem 2	Problem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?

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Guide 6b

Frank sees that the math in the problems is similar because the problems all include 32 feet of ribbon. Explain another way the math in the problems is similar, or a way the math in the problems is different. (Do not state that the problems contain the same numbers.)

b.

The proplems are a start formil-4 So they hold the same amount. I contration \c. Page 9 GO ON TO THE NEXT PAGE. Page 23

Guide 6	Litho 8220
Total Content Points: 3	(3.OA.2(a), 3.OA.2(b), 3.OA.3)
Total Practice Points: 1	(MP4)

This response contains 3 correct equations, each with the correct answer for the given problem (3.OA.2(a), 3.OA.3). Only 2 diagrams are drawn, but both are correctly constructed to model the division situations (3.OA.2(b)). By indicating that the student is dividing by 8 through counting groups of 8 units and keeping a tally of how many groups there are, the diagram for Problem 1 is a correct example of modeling (MP4). None of the answers given are labeled, which shows a lack of attention to the context of the problems (no credit for MP2), and insufficient attention to precision in the response (no credit for MP6). The student has not provided a diagram for Problem 3, so the task is not completed (no credit for MP1). The explanation for Part B referring to the "fact family" does not demonstrate enough understanding of the structure of the problems to be considered an acceptable comparison (no credit for MP7).

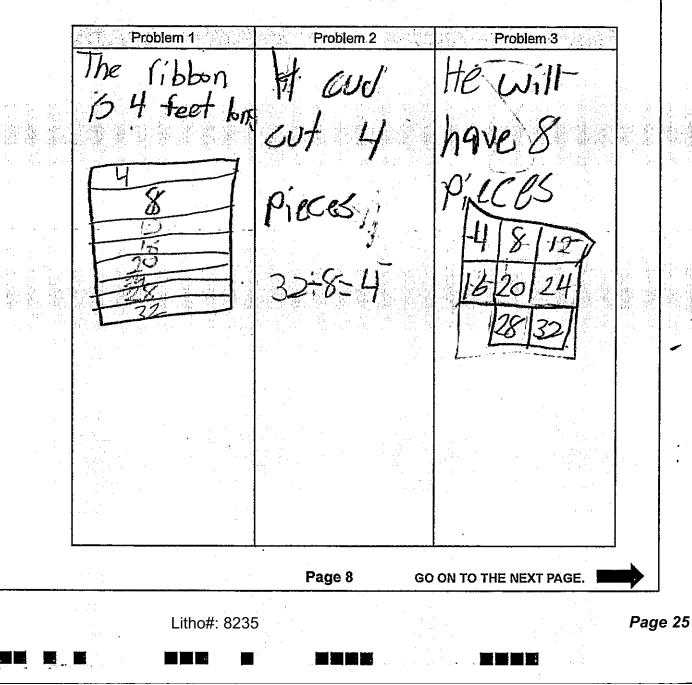
Total Awarded Points: 4 out of 8

## Guide 7a

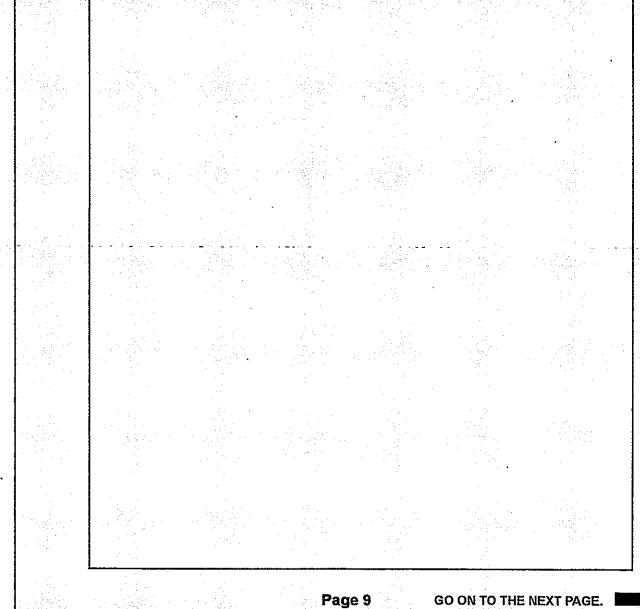
#### Task 2. Three Problems Task

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Problem 1 Problem 2 Problem 3		
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Frank sees that the math in the problems is similar because the problems all include b. 32 feet of ribbon. Explain another way the math in the problems is similar, or a way the math in the problems is different. (Do not state that the problems contain the same numbers.)



Guide 7	Litho 8235
Total Content Points: 2	(3.OA.2(b), 3.OA.3)
Total Practice Points: 1	(MP4)

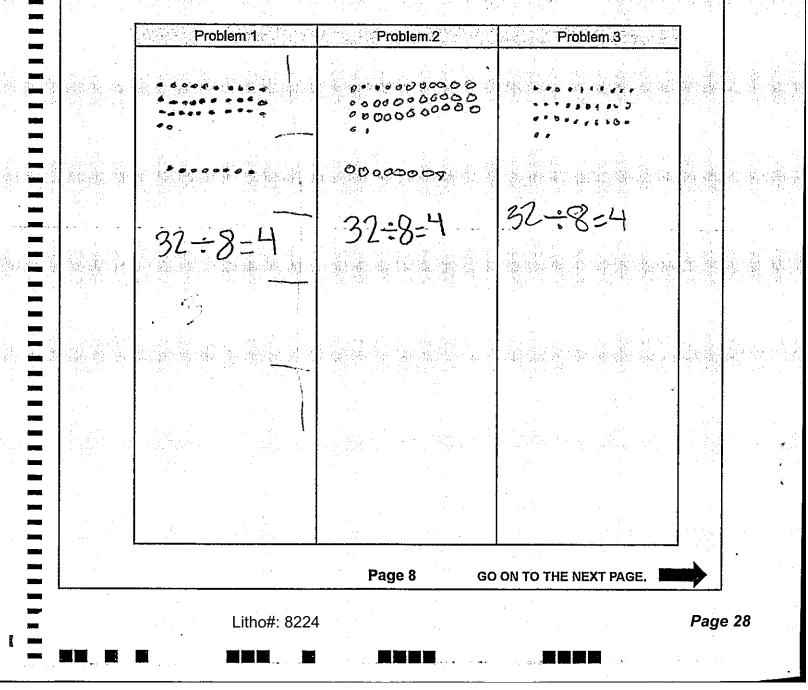
The two drawings in this response show the correct partitioning of the ribbon, including showing the lengths of the individual cut pieces (3.OA.2(b)). All three answers needed for Part A are given (3.OA.3), but the student has only included one equation (no credit for 3.OA.2(a)). The student creates two diagrams that show understanding of how to model division equations to determine the number of pieces cut (MP4). The student has not shown more than one equation, and has not attempted to make a comparison between the problems as asked for in Part B, so has not completed the task acceptably (no credit for MP1). Although the student has provided labels for the given answers, the lack of more than one equation does not allow the reader to clearly see the student's reasoning (no credit for MP2), and indicates a lack of attention to precision (no credit for MP6). There is no attempt to respond to Part B, so the student has not demonstrated a clear recognition of the structural similarities or differences between the three problems (no credit for MP7).

Total Awarded Points: 3 out of 8

Guide 8a

## Task 2. Three Problems Task

Problem 1	Problem 2	Problem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?



b. Frank sees that the math in the problems is similar because the problems all include
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Ò So, Franki sees that the math all include 32 feet of ribbon.

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Guide: 8		Litho 8224
Total Content Points:	2	(3.0A.2(a), 3.0A.3)

(MP4)

This response contains correct division equations  $(32 \div 8 = 4)$ , which indicates that the student understands how the 32 feet of ribbon can be divided (3.0A.2(a), 3.0A.3). The drawings shown do not correspond to the division equations (no credit for 3.0A.2(b)). The student has created two division equations that show understanding of how to determine the number of pieces to be cut (MP4). The student has not created acceptable diagrams, nor provided an explanation stating how the math problems are similar, so has not completed the task (no credit for MP1). The lack of acceptable diagrams does not allow the reader to clearly see the student's reasoning (no credit for MP2), and indicates a lack of precision (no credit for MP6). The incomplete response to Part B, which only restates a portion of the prompt, does not demonstrate a clear understanding of the structure of the problems (no credit for MP7).

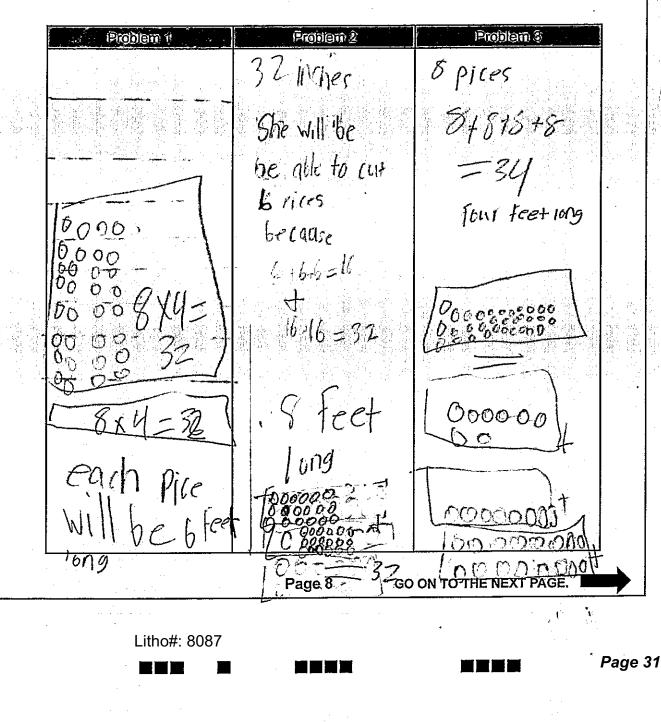
Total Awarded Points: 3 out of 8

Total Practice Points: 1

## Guide 9a

## Task 2. Three Problems Task

Roden (	Problem 2	Problem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?



## Guide 9b

Frank sees that the math in the problems is similar because the problems all include 32 feet of ribbon. Explain another way the math in the problems is similar, or a way the math in the problems is different. (Do not state that the problems contain the same numbers.)

b.

518 1818=32 6 tect long SNY=32 for each pice 2 6+6+6=16 16+16=32 Steet long for each pile 8 Pices four Feet long -BAX4=3=34 8+32=4 GO ON TO THE NEXT PAGE. Page 9

Guide 9	Litho 8087

Content Points: 1 (3.OA.2(b))

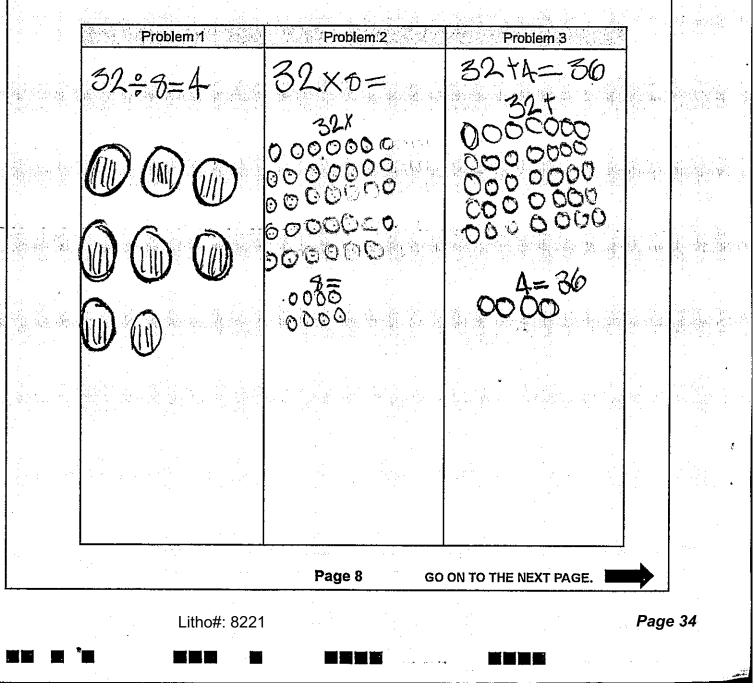
Practice Points: 0

This response contains a diagram that shows eight rows of four, indicating that the student understands how the 32 feet of ribbon can be divided (3.OA.2(b)). The student writes only one multiplication equation (8 x 4 = 32) (no credit for 3.OA.2(a)), and two of the answers given are incorrect (no credit for 3.OA.3). The student does not attempt to address Part B of the task, so the task is not completed (no credit for MP1). The work shown by the student matches neither the answers given nor the context of the problems (no credit for MP4, no credit for MP2), and not all of the calculations shown are correctly solved (no credit for MP6). As there is no attempt to respond to the second part of the task, the student has not demonstrated an understanding of the structure of the given problems (no credit for MP7).

Total Awarded Points: 1 out of 8

### Task 2. Three Problems Task

Problem 1	Problem 2	Problem 3
Jennifer has 32 feet of ribbon. She wants to cut it into 8 equal pieces. How long will each piece be?	Lucy has 32 feet of ribbon. She wants to cut it into pieces that are exactly 8 feet long. How many pieces can she cut?	Frank has 32 feet of ribbon. He wants to cut pieces of ribbon that are 4 feet long. How many pieces will Frank have?



b. Frank sees that the math in the problems is similar because the problems all include
 32 feet of ribbon. Explain another way the math in the problems is similar, or a way the math in the problems is different. (Do not state that the problems contain the same numbers.)

32=32=1

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Guide 10

#### Litho 8221

Total Content Points: 0

Total Practice Points: 0

This response contains only one correct equation and one correct drawing (Problem 1). There is not enough evidence that the student understands how to solve the problems based on the one correct drawing and equation, since the student's approach to the other similar problems is incorrect (no credit for 3.OA.2(a), no credit for 3.OA.2(b)). The student only finds one correct answer (no credit for 3.OA.3). The approach taken to find the answers is not valid, and the work shown does not indicate that the student understands the context of the problems (no credit for MP2), or how to model their solutions (no credit for MP4). The answer given in Part B does not show an understanding of the structure of the given problems (no credit for MP7). One of the equations attempted has no answer (no credit for MP6), and despite the correct work shown for Problem 1, the approaches the student takes to solve Problems 2 and 3 are too incorrect to show having made sense of the task (no credit for MP1).

Total Awarded Points: 0 out of 8