

Tennessee Comprehensive Assessment Program / Mathematics

TCAP/CRA PILOT 2012



Task 2 : More or Less than One? Scoring Guide

Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \underline{\hspace{2cm}}$



Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \underline{\hspace{2cm}}$



2. More or Less Than One Task Scoring Guide

The CCSS for Mathematical Content (2 points)

- 4.NF.2 Student compares the sum of the listed fractions to one whole using a visual model, and demonstrates or explains why Problem A is greater than one and Problem B is less than one. _____
- 4.NF.3a Drawing or equation indicates that the student recognizes that the three amounts are being combined to form one amount. _____
- Indicates the sum of two amounts and determines the remaining amount needed to sum to one whole.
 - Indicates the sum of the three amounts.

Total Content Points _____

The CCSS for Mathematical Practices (4 points)

- MP1 Student uses a diagram to compare the sums of Problems A and B to one whole, and student perseveres in determining that one sum is greater than one and one sum is less than one. _____
- (MP1: Make sense of problems and persevere in solving them.)
- MP3 Student presents a convincing argument to explain why the solution to Problem A is greater than one and the solution to Problem B is less than one whole. The student may: _____
- Show $\frac{1}{2}$ and $\frac{1}{4}$ of the figure shaded and state that $\frac{1}{3}$ cannot be shaded because only $\frac{1}{4}$ remains; or shade $\frac{1}{2}$ and $\frac{1}{3}$ and indicate that $\frac{1}{4}$ cannot be shaded because only $\frac{1}{6}$ remains and this is less than $\frac{1}{4}$.
 - Show $\frac{1}{3}$ and $\frac{1}{4}$ of the figure shaded and indicate that $\frac{1}{5}$ is less than both of these; therefore, the total sum is less than one whole.
- (MP3: Construct viable arguments and critique the reasoning of others.)
- MP4 Diagrams and equations indicate an awareness of the sizes of the fractions in comparison to each other. _____
- (MP4: Model with mathematics.)

MP6 Indicates more or less than a whole. _____

- Clearly indicates that the fractions in Problem A add up to more than one whole.
- Clearly indicates that the fractions in Problem B add up to less than one whole.

(MP6: Attend to precision.)

Total Practice Points _____

Total Awarded Points _____

The CCSS for Mathematical Content Addressed in This Task

Extend understanding of fraction equivalence and ordering.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

4.NF.3a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

The CCSS for Mathematical Practices*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*Gray text indicates Mathematical Practices not addressed in this task.

Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} =$ _____

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 1 \frac{1}{12}$

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} =$ _____

$\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$

Guide 1

Litho 40197

Total Content Points: 2 (4.NF.2, 4.NF.3a)

Total Practice Points: 4 (MP1, MP3, MP4, MP6)

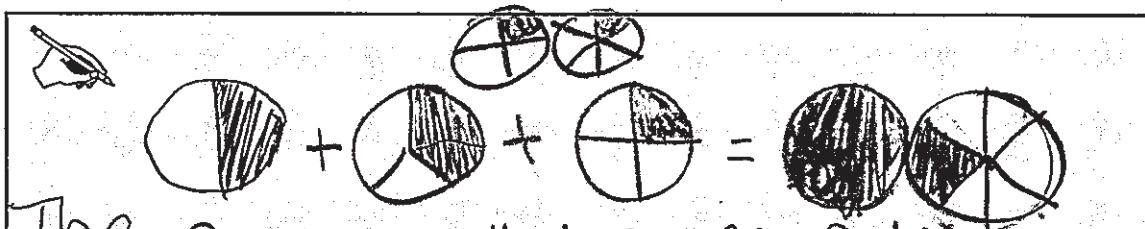
The student compares the sum of the fractions to one whole using a visual model, and explains why the solution to Problem A is greater than one and the solution to Problem B is less than one (4.NF.2). Drawings and equations indicate that the student recognizes that the three amounts are being combined to form one amount (4.NF.3a). The student uses visual models to compare the sums of Problems A and B to one whole and determines that one sum is greater than one, and one sum is less than one (MP1). The student uses diagrams to construct a convincing argument that the solution to Problem A is greater than one whole and the solution to Problem B is less than one whole (MP3). The use of diagrams and equations indicates an awareness of the sizes of the fractions in comparison to each other (MP4). The equations clearly and accurately demonstrate that the solution to Problem A is more than one whole and the solution to Problem B is less than one whole (MP6).

Total Awarded Points: 6 out of 6

Task 2. More or Less than One? Task

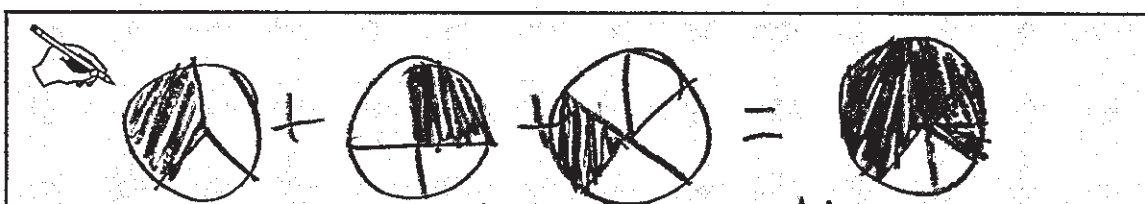
Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \underline{\hspace{2cm}}$



The sum will be more than one because $\frac{1}{2} + \frac{1}{3}$ needs $\frac{1}{6}$ more to make a whole but you have $\frac{1}{4}$ which is bigger than $\frac{1}{6}$ if you draw a picture.

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \underline{\hspace{2cm}}$



The sum would be less than one because $\frac{1}{4} + \frac{1}{5}$ is smaller than $\frac{1}{2}$ and $\frac{1}{3}$ is smaller than $\frac{1}{2}$.

Guide 2

Litho 40327

Total Content Points: 2 (4.NF.2, 4.NF.3a)

Total Practice Points: 3 (MP1, MP3, MP4)

The student compares the sum of the fractions to one whole using a visual model and explains why the solution to Problem A is greater than one and the solution to Problem B is less than one (4.NF.2). The student recognizes that the three amounts are being combined to form one amount (4.NF.3a). Visual models are used to compare the sums of Problems A and B to one whole and show that one sum is greater than one and one sum is less than one, with written explanations provided to clarify the student's reasoning (MP1). The student constructs a viable argument to convince the reader that the solution to Problem A is greater than one whole and the solution to Problem B is less than one whole (MP3). The use of diagrams and equations indicate an awareness of the sizes of the fractions in comparison to each other (MP4). In Problem A, the diagram representing the sum is divided and shaded into sixths rather than twelfths, indicating a lack of precision (no credit for MP6).

Total Awarded Points: 5 out of 6

Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \underline{\hspace{2cm}}$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 1\frac{1}{12}$

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \underline{\hspace{2cm}}$

$\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$

Guide 3

Litho 40367

Total Content Points: 2 (4.NF.2, 4.NF.3a)

Total Practice Points: 2 (MP1, MP4)

The student compares of the sum of the fractions to one whole using a visual model and demonstrates why the solution to Problem A is greater than one and the solution to Problem B is less than one (4.NF.2). The student demonstrates the recognition that the three amounts are being combined to form one amount (4.NF.3a). The response contains visual models comparing the sums of Problems A and B to one whole, indicating that one sum is greater than one and one sum is less than one (MP1). The use of diagrams and equations indicate an awareness of the sizes of the fractions in comparison to each other (MP4). Although the shaded circle shown in the explanation for Problem A might be a reference to one whole, the work for Problem B does not include a specific comparison of fractional parts to one whole, and consequently shows a lack of precision (no credit for MP6). Similarly, while some elements of an argument are present, the student does not include enough specific evidence to prove why the solution to Problem A is more than one whole and the solution to Problem B is less than one whole (no credit for MP3).

Total Awarded Points: 4 out of 6

Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{1}{6}$

The diagram shows three circles with different shading patterns: one with vertical lines, one with diagonal lines, and one with horizontal lines. Below these is a fourth circle that is completely scribbled out. To the right of the circles is an equals sign followed by the fraction $\frac{1}{8}$. There is also a small drawing of a hand holding a pencil in the top left corner of the box.

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{2}{3}$

The diagram shows three circles with different shading patterns: one with vertical lines, one with diagonal lines, and one with horizontal lines. Below these is a fourth circle that is completely scribbled out. To the right of the circles is an equals sign. There is also a small drawing of a hand holding a pencil in the top left corner of the box.

Guide 4

Litho 40030

Total Content Points: 2 (4.NF.2, 4.NF.3a)

Total Practice Points: 2 (MP1, MP4)

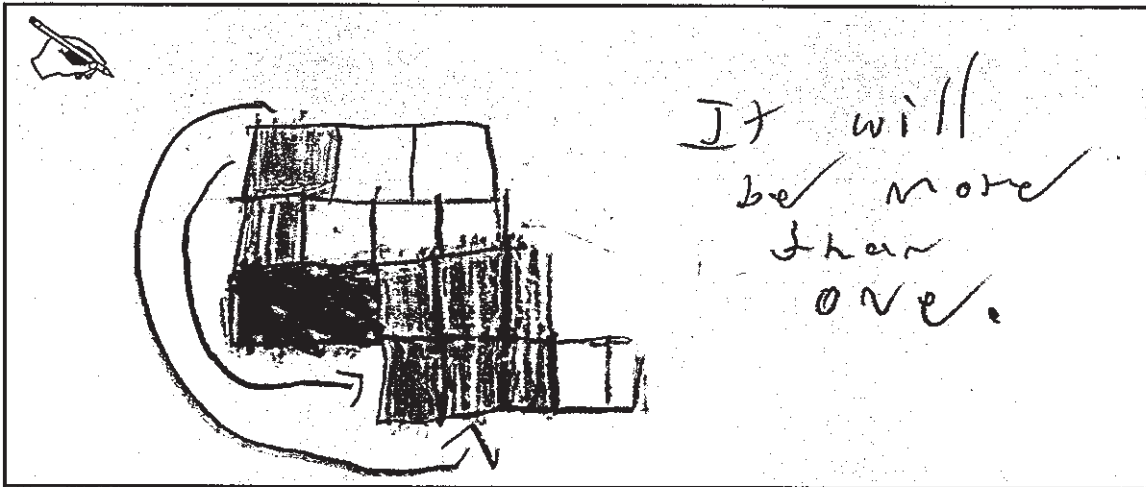
This response demonstrates that the student recognizes the three amounts are being combined to form one amount (4.NF.3a). The student uses visual models to show the size of the listed fractions and another visual model to show the total, indicating the relation of the sum of the fractions to one whole (4.NF.2, MP1). The student does not present a complete mathematical argument to explain to prove that the solution to Problem A is greater than one and the solution to Problem B is less than one (no credit for MP3). The use of diagrams indicates an awareness of the sizes of the fractions in comparison to each other (MP4). The student's work, both for Problem A and Problem B, does not include specific, nonvisual comparisons of the sum of the fractions to one whole, does not label all the diagrams, and includes incorrect sums, all of which indicate a lack of attention to precision (no credit for MP6).

Total Awarded Points: 4 out of 6

Task 2. More or Less than One? Task

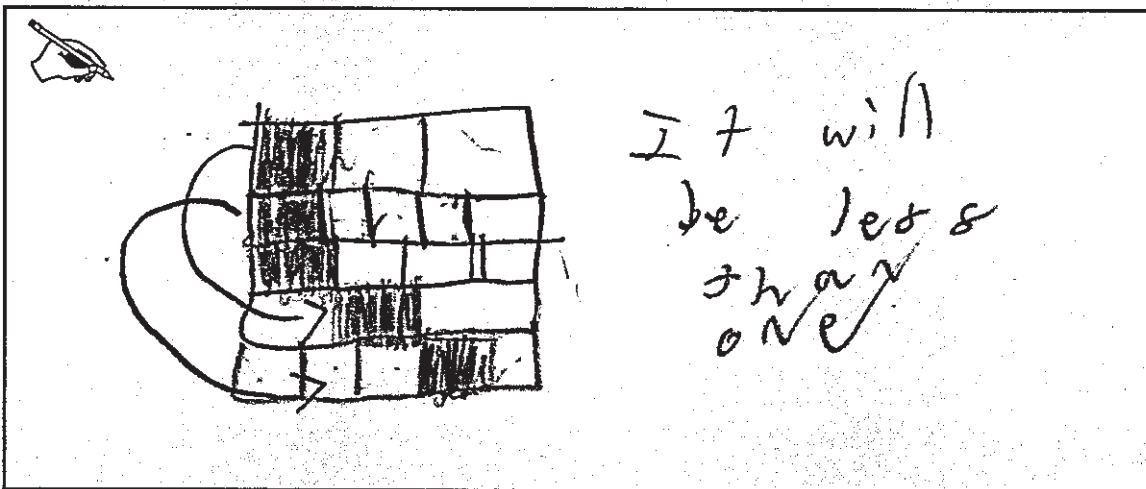
Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} =$ _____



The diagram shows a 2x3 grid. The top row has 2 shaded cells. The bottom row has 3 shaded cells. A large arrow points from the shaded area to the right. To the right of the grid, the handwritten text reads: "It will be more than one."

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} =$ _____



The diagram shows a 3x4 grid. The first column has 3 shaded cells. The second column has 1 shaded cell. The third column has 1 shaded cell. The fourth column has 0 shaded cells. A large arrow points from the shaded area to the right. To the right of the grid, the handwritten text reads: "It will be less than one."

Guide 5

Litho 40236

Total Content Points: 2 (4.NF.2, 4.NF.3a)

Total Practice Points: 1 (MP1)

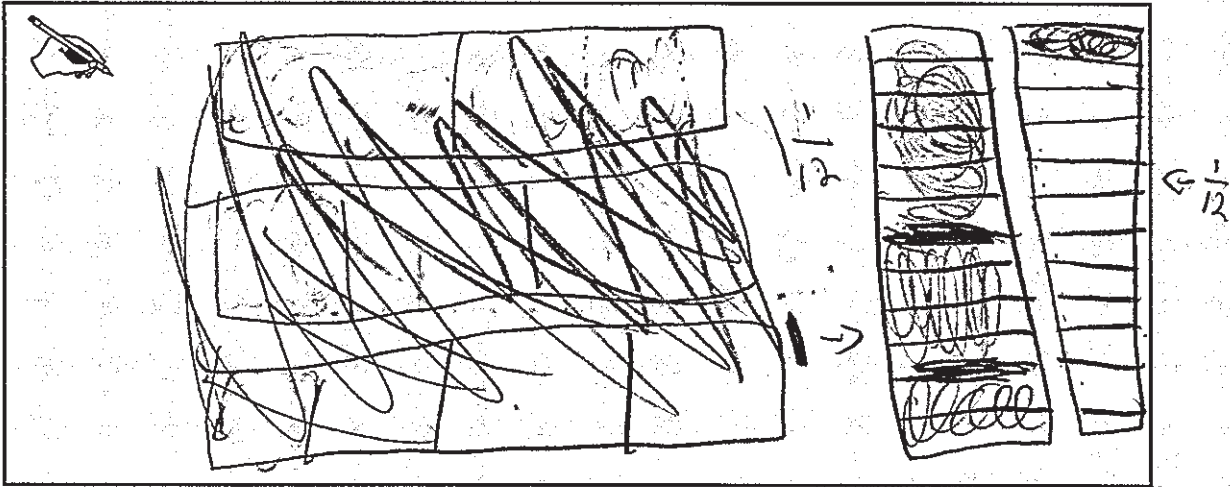
The student compares the sum of the fractions to one whole using a visual model and explains why the solution to Problem A is greater than one and the solution to Problem B is less than one (4.NF.2). The student demonstrates recognition that the three amounts are being combined to form one amount (4.NF.3a). The student uses visual models to compare the sums of Problems A and B to one whole and determines that one sum is greater than one and one sum is less than one (MP1). However, the student's diagram is both unclear and unlabeled. Therefore, it does not represent a viable mathematical argument (no credit for MP3), nor does it indicate precision (no credit for MP6). Although the student attempts to model with mathematics, his diagram for Problem A is confusing and does not clearly show an awareness of the fractions in comparison to each other (no credit for MP4).

Total Awarded Points: 3 out of 6

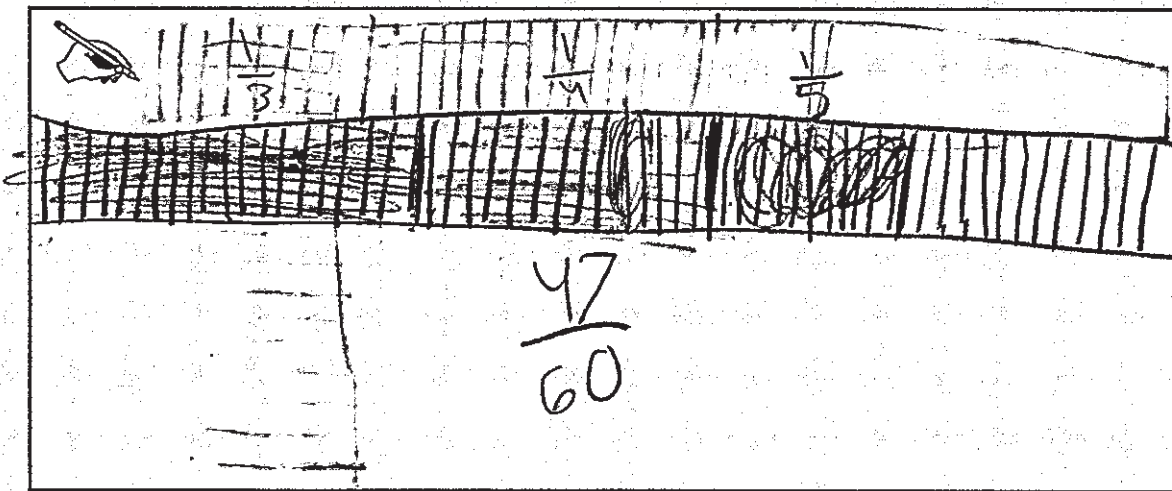
Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \underline{\hspace{2cm}}$



Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \underline{\hspace{2cm}}$



Guide 6

Litho 40252

Total Content Points: 2 (4.NF.2, 4.NF.3a)

Total Practice Points: 1 (MP1)

The student compares the sum of the fractions to one whole using a visual model and explains why the solution to Problem A is greater than one and the solution to Problem B is less than one (4.NF.2). The student demonstrates recognition that the three amounts are being combined to form one amount (4.NF.3a). The student uses visual models to compare the sums of Problems A and B to one whole and determines that one sum is greater than one and one sum is less than one (MP1). The student's mathematical argument is incomplete since no conclusion is explicitly drawn (no credit for MP3). The student's model for Problem A is not labeled. Although the student appears to have found a common denominator to make adding the fractions simpler, there is no specific work or explanation linking any of the given unit fractions to the model drawn (no credit for MP4). The student's work, for both Problem A and Problem B, does not include a specific nonvisual comparison of the sum of the fractions to one whole (no credit for MP6).

Total Awarded Points: 3 out of 6

Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \underline{\hspace{2cm}}$

More, because I know that $\frac{1}{2} + \frac{1}{4}$ is more than a half.

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \underline{\hspace{2cm}}$

Less because you can tell that it will be smaller than a whole.

Guide 7

Litho 40016

Total Content Points: 1 (4.NF.3a)

Total Practice Points: 1 (MP4)


This student indicates recognition that the three amounts are being combined to form one amount (4.NF.3a), but her visual model does not compare the combined fractions to one whole (no credit for 4.NF.2). The use of diagrams indicates an awareness of the sizes of the fractions in comparison to each other (MP4). However, the diagrams do not show a clear comparison of the sums of the fractions to one whole (no credit for MP1). The student's mathematical argument is incomplete in Problem A and little more than an assertion in Problem B (no credit for MP3). The student's work does not present clear, specific logic to illustrate why the sums are more or less than a whole. Since the conclusions reached are general and unsubstantiated ("more"; "smaller"), attention to precision is lacking (no credit for MP6).

Total Awarded Points: 2 out of 6


Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

$$\text{Problem A: } \frac{\overset{6}{\cancel{6}}}{\underset{12}{\cancel{12}}} + \frac{\overset{4}{\cancel{4}}}{\underset{12}{\cancel{12}}} + \frac{\overset{3}{\cancel{3}}}{\underset{12}{\cancel{12}}} = \frac{1}{12} \text{ or } \frac{13}{12}$$

 What you would have to do is to find a common denominator like this. What can 2, 3, and 4 all go into... 12. So $\frac{1}{2} \times 6 = \frac{6}{12}$, $\frac{1}{3} \times 4 = \frac{4}{12}$ and $\frac{1}{4} \times 3 = \frac{3}{12}$ and $\frac{6}{12} + \frac{3}{12} + \frac{4}{12} = \frac{13}{12}$ or $\frac{1}{12}$

$$\text{Problem B: } \frac{\overset{20}{\cancel{20}}}{\underset{60}{\cancel{60}}} + \frac{\overset{15}{\cancel{15}}}{\underset{60}{\cancel{60}}} + \frac{1}{5} = \frac{47}{60}$$

 You do the same as problem A. You find the common denominator. So $\frac{1}{3} \times 20 = \frac{20}{60}$, $\frac{1}{4} \times 15 = \frac{15}{60}$, $\frac{1}{5} \times 12 = \frac{12}{60}$. Then you add $\frac{20}{60} + \frac{15}{60} + \frac{12}{60} = \frac{47}{60}$

Guide 8

Litho 40232

Total Content Points: 1 (NF.3a)

Total Practice Points: 0

This response demonstrates the student's recognition that the three amounts are being combined to form one amount (4.NF.3a). The student's work does not include a visual model (no credit for 4.NF.2, no credit for MP1). Specific visual comparisons of the sums of the fractions to one whole are missing for both Problem A and Problem B, and while the work shown to find the common denominators is incorrect, the student does indicate awareness of the sizes of the fractions in comparison to each other. However, without visual diagrams to model the fractions, the equations are considered inappropriate modeling for this task (no credit for MP4). The student does not create a visual mathematical argument (no credit for MP3). There is an overall lack of detail and attention to precision (no credit for MP6).

Total Awarded Points: 1 out of 6

Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} =$ _____

The sum is more than one.

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} =$ _____

The sum is less than one.

Total Content Points: 0

Total Practice Points: 1 (MP4)

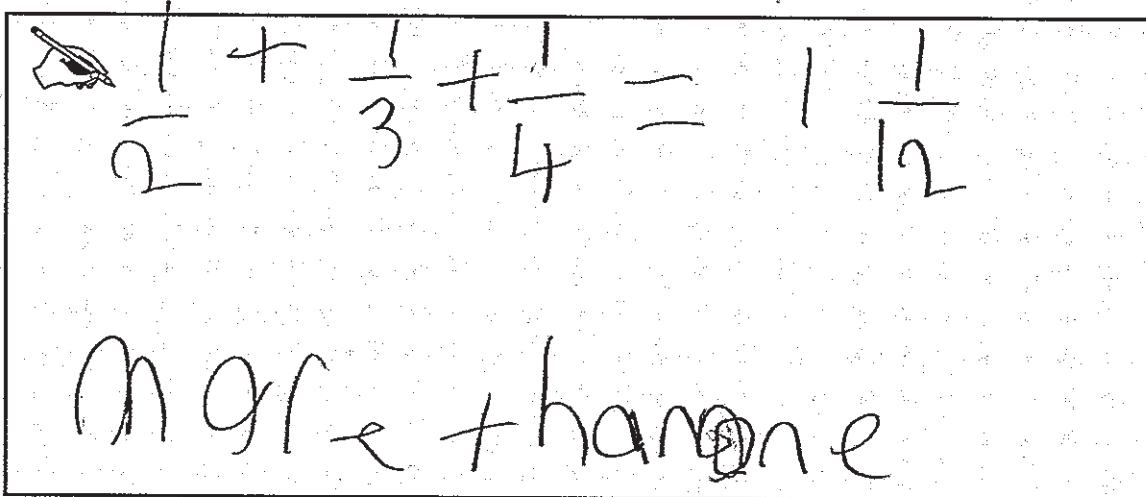
This response does not include a visual comparison of the given fractions to one whole (no credit for 4.NF.2). The student does not indicate that the three amounts are being combined to form one amount (no credit for 4.NF.3a). The student uses diagrams indicating awareness about the sizes of the fractions in comparison to each other (MP4). The student's diagram does not include a comparison of the sums of the given fractions to one whole (no credit for MP1), nor is a mathematical argument constructed to prove that the combined fractions represent more or less than one whole (no credit for MP3). There is an overall lack of detail and attention to precision (no credit for MP6).

Total Awarded Points: 1 out of 6

Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

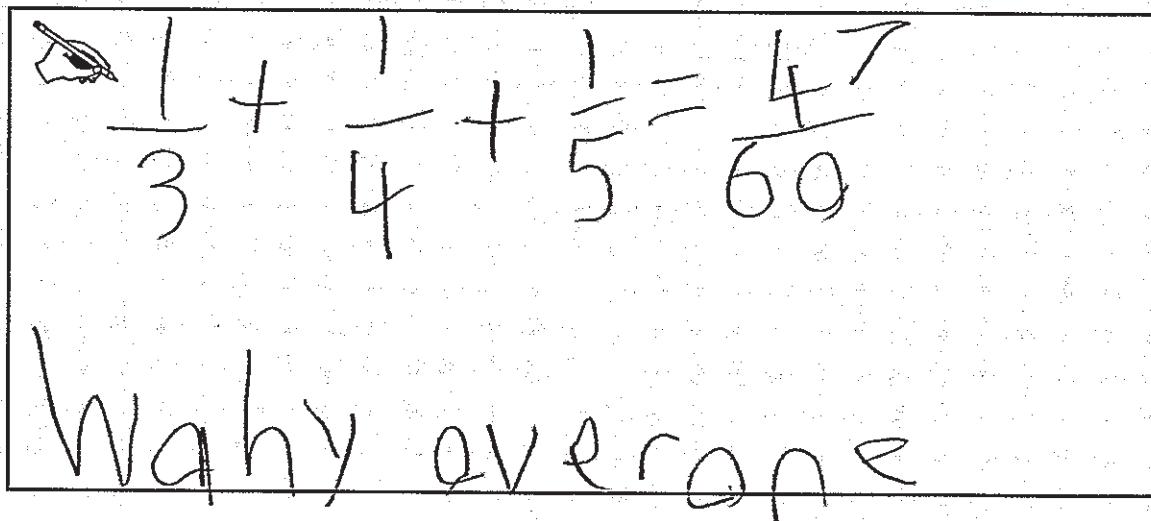
Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \underline{\hspace{2cm}}$



$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 1 \frac{1}{12}$

More than one

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \underline{\hspace{2cm}}$



$\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{4}{60}$

Way over one

Guide 10

Litho 40355

Total Content Points: 1 (4.NF.3a)

Total Practice Points: 0

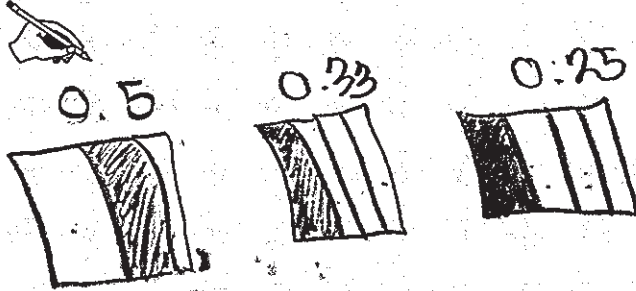
This response demonstrates the student recognizes the three amounts are being combined to form one amount (4.NF.3a). It does not include visual models (no credit for 4.NF.2, no credit for MP1, no credit for MP3). There is not enough work shown to indicate that the student understands the sizes of the fractions in comparison to one another (no credit for MP4). There is not enough appropriate work to demonstrate attention to precision (no credit for MP6).

Total Awarded Points: 1 out of 6

Task 2. More or Less than One? Task


Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \underline{\frac{1}{12}}$



$\frac{1}{2}$ is less than all of them because 3 is bigger 4 is bigger so $\frac{1}{4}$ is smallest

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \underline{\hspace{2cm}}$



$\frac{1}{5}$ is the smallest one because $\frac{1}{3}$ is 0.33 and $\frac{1}{4}$ is 0.25 and $\frac{1}{5}$ is 0.20 so that's the smallest one.

Total Content Points: 0

Total Practice Points: 0

The student does not indicate that the three amounts are being combined (no credit for 4.NF.3a) and does not compare the sum of the fractions to one whole (no credit for 4.NF.2, no credit for MP1, no credit for MP3). While the visual models do indicate some awareness of the sizes of the fractions in comparison to each other, the written explanation contains errors and contradicts the drawn models (no credit for MP4). The visual models are inaccurate, which contributes to an overall lack of detail and attention to precision (no credit for MP6).

Total Awarded Points: 0 out of 6

Task 2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum will be more or less than one.

Problem A: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \underline{1\frac{1}{12}}$



You know the the sum will be more than 1 because just

$$4 + 3 + 2 = 9$$

Problem B: $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \underline{\frac{47}{60}}$



You know the sum will be more because just $5 + 4 + 3 = 12$

Total Content Points: 0

Total Practice Points: 0

This response demonstrates a lack of understanding about the task. The student provides correct solutions, but the explanations of adding denominators to arrive at whole numbers contradict the solutions and show misunderstanding of the process of combining fractions to create one whole (no credit for 4.NF.3a). The student does not use any visual models and does not compare either the fractions or the sums to one whole (no credit for 4.NF.2, no credit for MP1, no credit for MP3, no credit for MP4). There is not enough appropriate work to demonstrate attention to precision (no credit for MP6).

Total Awarded Points: 0 out of 6