Tennessee Comprehensive Assessment Program / Mathematics

TCAP/CRA PILOT 2012



Task 4 : The Cake Shop Scoring Guide

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Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys $\frac{2}{6}$ of each type of cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.





GO ON TO THE NEXT PAGE.



4. The Cake Shop Task Scoring Guide

The CCSS for Mathematical Content (2 points)

4.NF.4a Student recognizes that
$$\frac{2}{6}$$
 is $\frac{1}{6} + \frac{1}{6}$ or 2 x $\frac{1}{6}$.

4.NF.4c Student identifies the product of 4 x $\frac{2}{6}$. Work may show:

- $\frac{2}{6}$ added four times to arrive at $\frac{8}{6}$ (or $1 \frac{2}{6}$ or $1 \frac{1}{3}$).

$$- \frac{2}{6} \times 4 = \frac{8}{6} .$$

Total Content Points

The CCSS for Mathematical Practices (5 points)

MP1 Student makes sense of and completes all aspects of the task.

(MP1: Make sense of problems and persevere in solving them.)

MP2 Student writes equations and provides labels that indicate the meaning of the amounts as they relate to the context of the problem.

(MP2: Reason abstractly and quantitatively.)

(MP3: Construct viable arguments and critique the reasoning of others.)

MP4 Representation indicates an understanding of the amount of cake bought by Lisa. The equation indicates an understanding of alternative ways of thinking about $4 \times \frac{2}{6}$.

 $\frac{1}{6}$

(MP4: Model with mathematics.)

MP6 Accurate and precise work is shown.

- Work indicates the product of 4 x $\frac{2}{6}$ as $\frac{8}{6}$ or 1 $\frac{2}{6}$ or 1 $\frac{1}{3}$.
- Work identifies 8 x $\frac{1}{6}$ as the equivalent expression related to 4 x $\frac{2}{6}$.

(MP6: Attend to precision.)

Total Practice Points _____

Total Awarded Points _____

The CCSS for Mathematical Content Addressed in This Task

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

- 4.NF.4a Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.
- 4.NF.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

The CCSS for Mathematical Practices*

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

*Gray text indicates Mathematical Practices not addressed in this task.

Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

Guide 1a

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.



Page 10 GO ON TO THE NEXT PAGE.





Litho 40159

Total Content Points: 2 (4.NF.4a, 4.NF.4c)

Total Practice Points: 5 (MP1, MP2, MP3, MP4, MP6)

The student correctly identifies the product of $4 \times \frac{2}{6}$ by adding $\frac{2}{6}$ four times (4.NF.4c). The student clearly recognizes that 4 pieces equal to $\frac{2}{6}$ are equivalent to 8 pieces equal to $\frac{1}{6}$ (4.NF.4a). The student makes sense of and completes all aspects of the task (MP1). The response contains equations and the labels, such as " $1\frac{2}{6}$ of cake," that indicate the meaning of the amounts as they relate to the context of the problem (MP2). The student provides a convincing argument by stating, " $8 \times \frac{1}{6}$ is lik saying you have 8 peices of cake that are $\frac{1}{6}$." Further proof of the validity of this statement can be seen in the student's addition problem (MP3). The response includes a drawing representing the amount of cake bought by Lisa (MP4), and the work is accurate and precise (MP6).

Total Awarded Points: 7 of 7

Guide 2a

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.







Litho 40369

Total Content Points: 2 (4.NF.4a, 4.NF.4c)

Total Practice Points: 5 (MP1, MP2, MP3, MP4, MP6)

The student correctly identifies the product of $4 \times \frac{2}{6}$ by adding $\frac{2}{6}$ four times (4.NF.4c). The student clearly recognizes that 4 pieces equal to $\frac{2}{6}$ are equivalent to 8 pieces equal to $\frac{1}{6}$ (4.NF.4a). The student makes sense of all aspects of the task (MP1). The student constructs equations correctly and provides labels, such as "1 and $\frac{2}{6}$ of cake," that indicate the meaning of the amount as it relates to the context of the problem (MP2). The student constructs an argument convincing the reader that $4 \times \frac{2}{6}$ is the same as

 $8 \times \frac{1}{6}$ by stating, "you split one number in half and doubled the other." The example provided by the student is further proof of the validity of this concept (MP3). The response includes a representation of the amount of cake bought by Lisa (MP4), and the work is accurate and precise (MP6).

Total Awarded Points: 7 of 7

Guide 3a

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.





Lisa claims:

I can figure out the number of pieces that I have by thinking about the pieces that are each $\frac{1}{6}$ of the cake.

b. When Lisa thinks about the cake this way, she writes $4 \times 2 \times \frac{1}{6}$ of a cake. Explain why Lisa can think about the cake this way. Use words or diagrams.

Page 11

Litho#: 40312

Because 442=876=13.



40312 Page 13

Litho 40312

Total Content Points: 2 (4.NF.4a, 4.NF.4c)

Total Practice Points: 2 (MP2, MP3)

The student correctly identifies the product of $4 \times \frac{2}{6}$ by adding $\frac{2}{6}$ four times to get an answer of $1\frac{2}{6}$ (4.NF.4c). The student demonstrates that 8 pieces, each of which is $\frac{1}{6}$ in size, are equivalent to 4 pieces, each of which is $\frac{2}{6}$ in size, by stating " $4 \times 2 = 8 \times \frac{1}{6} = 1\frac{1}{3}$ " (4.NF.4a). The student writes an equation and provides a diagram with labels to indicate the meaning of the amounts as they relate to the context of the problem (MP2). The response constructs a viable argument indicating that $4 \times \frac{2}{6}$ is the same as $8 \times \frac{1}{6}$ (MP3). The response, however, has a flawed equation, $4 \times 2 = 8 \times \frac{1}{6} = 1\frac{1}{3}$ (no credit for MP4), and lacks a clear explanation in part b, which demonstrates a lack of precision (no credit for MP6) and an inability to make sense of all aspects of the task (no credit for MP1).

Total Awarded Points: 4 of 7

Guide 4a

Task 4. The Cake Shop Task

equals 2,

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Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.

Since she bought 4 and each piece

4+2=18



Guide 4b Lisa claims: I can figure out the number of pieces that I have by thinking about the pieces that are each $\frac{1}{6}$ of the cake. When Lisa thinks about the cake this way, she writes $4 \times 2 \times \frac{1}{6}$ of a cake. Explain why b. Lisa can think about the cake this way. Use words or diagrams. Cake. Then she took the 2 from the numerator and changed it into a land kept the denomination a 6 so if you do When it was 4x3 = 12 and you can simplify it WORK IF YOU Page 11 HAVE TIME 40288 Page 16 Litho#: 40288

Litho 40288

Total Content Points: 2 (4.NF.4a, 4.NF.4c)

Total Practice Points: 2 (MP2, MP3)

The student correctly multiplies a fraction, $\frac{2}{6}$, by a whole number, 4, to get a correct answer of $1\frac{2}{6}$ (4.NF.4c). The student understands that $\frac{2}{6}$ is equal to doubling $\frac{1}{6}$ (4.NF.4a). The student includes an equation and provides labels, such as " $1\frac{2}{6}$ of cake," that indicate the meaning of the amounts as they relate to the context of the problem (MP2). The student constructs a viable argument recognizing that $\frac{1}{6}$ is half of $\frac{2}{6}$ (MP3). The response, however, does not include diagrams (no credit for MP4), indicating a failure to persevere in solving all aspects of the task (no credit for MP1). The student's explanation in part b is unclear and lacks precision (no credit for MP6).

Total Awarded Points: 4 of 7

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

Guide 5a

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.



 Page 10
 GO ON TO THE NEXT PAGE.

 Litho#: 40231
 Page 18



Litho 40231

Total Content Points: 2 (4.NF.4a, 4.NF.4c)

Total Practice Points: 1 (MP3)

The student correctly recognizes that the product of $4 \times \frac{2}{6}$ is $1\frac{2}{6}$ (4.NF4.c). The

student's work demonstrates the understanding that 8 pieces, each of which is $\frac{1}{6}$ in size, are equivalent to 4 pieces, each of which is $\frac{2}{6}$ in size, by stating " $4 \times 2 = 8 \times \frac{1}{6} = 1 \frac{2}{6}$ or $1 \frac{1}{2}$ it = the same thing" (4.NF.4a). The student provides a convincing argument to

3 support Lisa's claim (MP3). The response fails to provide labels to indicate the meaning of the amounts as they relate to the context of the problem (no credit for MP2). Additionally, the student does not write an equation for part a (no credit for MP4) and

constructs a flawed equation in part b, " $4 \times 2 = 8 \times \frac{1}{6} = 1 \frac{2}{6}$ " (no credit for MP6),

indicating a failure to make sense of and persevere in solving all aspects of the task (no credit for MP1).

Total Awarded Points: 3 of 7

Guide 6a

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.





GO ON TO THE NEXT PAGE.

Page 10



Total Content Points: 2 (4.NF.4a, 4.NF.4c)

Total Practice Points: 1 (MP3)

Although the product of $4 \times \frac{2}{6}$ is not stated in part a, the student indicates $\frac{2}{6}$ added 4 times and provides the correct answer of $1\frac{2}{6}$ in response to part b (4 .NF.4c). The student demonstrates the understanding that 8 pieces, each of which is $\frac{1}{6}$ in size, are equivalent to 4 pieces, each of which is $\frac{2}{6}$ in size, by stating, " $8 \times \frac{1}{6} = 1\frac{2}{6}$ which is equal to $\frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6}$ " (4.NF.4a). The student provides a convincing argument to support Lisa's claim and demonstrates that the product of $4 \times \frac{2}{6}$ is equivalent to $8 \times \frac{1}{6}$ (MP3). The response contains equations, but does not contain appropriate diagrams (no credit for MP4), and does not include the labels necessary to indicate the meaning of the amounts in the context of the problem (no credit for MP2). This indicates a lack of precision (no credit for MP6) and an inability to make sense of and persevere in solving all aspects of the task (no credit for MP1).

Total Awarded Points: 3 of 7

Chacol te

a.

Guide 7a

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

vinilla

Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.

Chely

alrond





b.



I can figure out the number of pieces that I have by thinking about the pieces that are each $\frac{1}{6}$ of the cake.

When Lisa thinks about the cake this way, she writes $4 \times 2 \times \frac{1}{6}$ of a cake. Explain why Lisa can think about the cake this way. Use words or diagrams.

It also equills 12



Page 25

40351



Litho#: 40351

Guide 7 Litho 40351

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 2 (MP2, MP4)

The student correctly identifies the product of $4 \times \frac{2}{6}$ by adding $\frac{2}{6}$ four times to arrive at $1\frac{2}{6}$ (4.NF.4c). The student's assertion that "it also equills $1\frac{2}{6}$ " fails to demonstrate an understanding of fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ (no credit for 4.NF.4a). The student writes an equation, draws a diagram, and correctly labels the diagram, indicating an understanding of the amount of cake bought by Lisa (MP2, MP4). There is no evidence to suggest that the student understands that 8 pieces that are each $\frac{1}{6}$ in size are equivalent to 4 pieces that are each $\frac{2}{6}$ in size (no credit for MP3). This indicates a lack of precision (no credit for MP6) and an inability to make sense of all aspects of the task (no credit for MP1).

Total Awarded Points: 3 of 7

Guide 8a

Task 4. The Cake Shop Task

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.

Lisq has 12 cake altogether.

Page 10 GO ON TO THE NEXT PAGE. Page 27 Litho#: 40029

2+2+2+2=12



Litho 40029

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 1 (MP2)

The student correctly identifies the product of $4 \times \frac{2}{6}$ by adding $\frac{2}{6}$ four times to arrive at $1\frac{2}{6}$ (4.NF.4c). The student's response to part b is inadequate because it simply reiterates Lisa's expression without recognizing that 4 pieces that are each $\frac{2}{6}$ in size are equivalent to 8 pieces that are each $\frac{1}{6}$ in size (no credit for 4.NF.4a). The student receives credit for writing an equation $(\frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6} = 1\frac{2}{6})$ and correctly labeling the meaning of the amount, " $1\frac{2}{6}$ cake," as it relates to the context of the problem (MP2). The student does not include a diagram, and there is no evidence to suggest that the student understands that 8 pieces that are each $\frac{1}{6}$ in size are equivalent to 4 pieces that are each $\frac{2}{6}$ in size (no credit for MP4, no credit for MP3). This indicates a lack of precision (no credit for MP6) and an inability to make sense of all aspects of the task (no credit for MP1).

Total Awarded Points: 2 of 7

Guide 9a

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.







Litho 40404

Total Content Points: 1(4.NF.4a)

Total Practice Points: 1 (MP3)

The student's diagram accurately shows four models of $\frac{2}{6}$. However, the attempt to label the diagram is inaccurate (no credit for MP2) and indicates some confusion with multiplication of a fraction by a whole number (no credit for 4.NF.4c). Although the correct answer of $1\frac{2}{6}$ is provided, correct work is not shown (no credit for MP4), and the simplification is incorrect (no credit for MP6). The student's work in part b of the task indicates that the student recognizes that 4 pieces that are each $\frac{2}{6}$ in size are equivalent to 8 pieces that are each $\frac{1}{6}$ in size (4.NF.4a). The student demonstrates that $4 \times \frac{2}{6}$ is the same as $8 \times \frac{1}{6}$ (MP3). The error in simplification and the lack of clarity in the response indicate an inability to make sense of all aspects of the task (no credit for MP1).

Total Awarded Points: 2 of 7

Guide 10a

Task 4. The Cake Shop Task

2/16 3/2/6

a.

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.

She has 12 ato gether



Lisa claims:

b.

I can figure out the number of pieces that I have by thinking about the pieces that are each $\frac{1}{6}$ of the cake.

When Lisa thinks about the cake this way, she writes $4 \times 2 \times \frac{1}{6}$ of a cake. Explain why Lisa can think about the cake this way. Use words or diagrams.

She con think that way be couse when I did it in the other question Igot the Bameanswer beit I did it a diffront -Way.



Page 34

40006

Litho#: 40006

Guide 10 Litho 40006

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 0

The student correctly identifies the product of $4 \times \frac{2}{6}$ (4.NF.4c). The student does not demonstrate an understanding of fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ in words or equations (no credit for 4.NF.4a). The student's response to part a contains an expression, not an equation (no credit for MP2), and lacks both a diagram and a description of Lisa's cake (no credit for MP4). Additionally, the response to part b of the task lacks clarity. There is no evidence to suggest that the student understands that 8 pieces that are each $\frac{1}{6}$ in size are equivalent to 4 pieces that are each $\frac{2}{6}$ in size (no credit for MP3). This indicates a lack of precision (no credit for MP6) and an inability to make sense of all aspects of the task (no credit for MP1).

Total Awarded Points: 1 of 7

Guide 11a

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.

Litho#: 40314

Page 10

GO ON TO THE NEXT PAGE.



Guide 11 Litho 40314

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 0

The student identifies the product of $4 \times \frac{2}{6}$ by adding $\frac{2}{6}$ four times as well as by multiplying (4.NF.4c). The student's explanation is incomplete because the student fails to recognize that the number of pieces needs to be doubled when $\frac{1}{6}$ -sized pieces are considered instead of $\frac{2}{6}$ (no credit for 4.NF.4a). The student did not make sense of and complete all aspects of the task (no credit for MP1). Although the student indicated the meaning of the amount as it relates to the context, " $1\frac{1}{3}$ of cake," no equations are given (no credit for MP2, no credit for MP4). The student repeats Lisa's expression as given in the task with no indication that 8 pieces that are each $\frac{1}{6}$ in size (no credit for MP3). The response does not include a diagram or equation and lacks precision (no credit for MP6).

Total Awarded Points: 1 of 7

Guide 12a

Task 4. The Cake Shop Task

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is $\frac{2}{6}$ of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.

The will have to of cake because times 4x2=8x1=87, which will be the numorator then times 8x6 which is 48.

Page 10

Litho#: 40417





GO ON TO THE NEXT PAGE.



Litho 40417

Total Content Points: 0

Total Practice Points: 0

The student's diagram accurately depicts $\frac{2}{6}$ of 4 cakes. However, the student's computations demonstrate no understanding of the multiplication of a fraction by a whole number (no credit for 4.NF.4c). Although there is some indication that the student recognizes that 8 pieces that are each $\frac{1}{6}$ in size are equivalent to 4 pieces that are each $\frac{2}{6}$ in size, the student shows a fundamental misunderstanding of the meaning of the denominator in relation to multiplication of fractions, and does not recognize the meaning of $\frac{a}{b}$ as it relates to $a \times \frac{1}{b}$ (no credit for 4.NF.4a, no credit for MP3). The calculations provided are incorrect (no credit for MP2, no credit for MP4), indicating a lack of precision (no credit for MP6) and an inability to make sense of all aspects of the task (no credit for MP1).

Total Awarded Points: 0 of 7