**Tennessee Comprehensive Assessment Program / Mathematics** 

# TCAP/CRA 2012-2013



# Task 3: Comparing Squares Task Full Scoring Guide

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Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

a. Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.



b. Write two different but equivalent ratios that compare the area of the larger square to the area of the smaller square. Explain how you found both ratios.

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# 3. Comparing Squares Task Scoring Guide

## The CCSS for Mathematical Content (3 points)

- 6.RP.1 The student uses ratios and/or ratio language to describe a relationship between the areas of the two squares.
- 6.RP.3 The student uses ratio and/or rate reasoning to find two different, but equivalent, ratios in any of the following ways (student may refer to diagram in explanation):

• by scaling, e.g.,  $\frac{144 \text{ sq. units}}{16 \text{ sq. units}} = \frac{9 \text{ sq. units}}{1 \text{ sq. unit}} = \frac{27 \text{ sq. units}}{3 \text{ sq. units}}$ , etc.

- by dividing or finding a unit rate, e.g.,  $\frac{144}{16} = 9$ ; therefore, the ratio is 144 square units to 16 square units, or 9 square units to 1 square unit.
- by using a proportion or proportional reasoning (e.g.,  $\frac{9 \text{ sq. units}}{1 \text{ sq. unit}} = \frac{x \text{ sq. units}}{9 \text{ sq. units}}$ ).
- by scaling up using a table.
- by describing the diagram, e.g., the ratio of little squares in the final square to little squares in the original square is 144:16, while the ratio of big squares in the final square to big squares in the original square is 9:1.
- 6.EE.1 The student writes and evaluates one or more correct expressions or equations containing exponents to represent the area of Kara's square. Work may include any of the following expressions, equations, and/or techniques for evaluation:
  - $A = 12^2 = 12 \times 12 = 144$  square units
  - $A = 3^2 \times 4^2 = 9 \times 16 = 144$  square units
  - $A = (4 \times 3) \times (4 \times 3) = (4 \times 3)^2 = 144$  square units
  - $A = 4^2 \times 3^2 = 16 \times 9 = 144$  square units
  - $A = 9(4 \times 4) = 9 \times 4^2 = 9 \times 16 = 144$  square units

Total Content Points \_\_\_\_\_

# The CCSS for Mathematical Practice (5 points)

MP1 The student completes all parts of the problem, including recognizing how to determine the area of the enlarged square and how to express the ratio of the areas.

(MP1: Make sense of problems and persevere in solving them.)

MP2 The student interprets "3 times longer and 3 times wider" as meaning multiplication of the length and width by 3, appropriately writes an expression scaling both the length and width of the square, contextualizes the results as the area of the larger square, and labels quantities.

(MP2: Reason abstractly and quantitatively.)

MP4 The student uses ratios, number sentences, proportions, expressions including a variable, and/or equations to model Kara's new square.

(MP4: Model with mathematics.)

MP6 The student writes expressions or equations using exponents accurately, correctly labels numerical quantities (e.g., 12<sup>2</sup> square units), and provides a carefully formulated explanation.

(MP6: Attend to precision.)

MP7 The student provides an explanation for how the second ratio is determined, and recognizes and uses the multiplicative structure of proportional relationships to find the second ratio.

(MP7: Look for and make use of structure.)

Total Practice Points \_\_\_\_\_

Total Awarded Points \_\_\_\_\_

# The CCSS for Mathematical Content Addressed in This Task

#### Understand ratio concepts and use ratio reasoning to solve problems.

- 6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
- 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

#### Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

#### The CCSS for Mathematical Practice\*

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

\*Gray text indicates Mathematical Practices that are not addressed in this task.

Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

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b.

Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

 Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.

The new square is 12 times 12 or 122. Just like 4x4 or 42 Area small Area big 16 144



Guide 1	Litho 11455
Total Content Points: 3	(6.RP.1, 6.RP.3, 6.EE.1)
Total Practice Points: 4	(MP1, MP2, MP4, MP7)

In this response the student uses ratios that describe a relationship between the areas of two

squares, such as "16 to 144;  $\frac{1}{9}$ " (6.RP.1). The student uses rate reasoning to find two different, but equivalent, ratios ("16:144; 1:9") (6.RP.3). The response provides one or more expressions containing exponents to represent the area of Kara's square ("12 times 12 or 12<sup>2</sup>; Area big 144") (6.EE.1). All parts of the problem are completed (MP1). The student understands 3 times longer and 3 times wider, uses 4 x 4 and 12 x 12, and writes equations scaling both the length and width of the square (MP2). The student uses ratios to model Kara's new square ("16:144; 12 times 12; 12<sup>2</sup>; area big 144") (MP4). The work provided is not labeled (no credit for MP6). The student provides an explanation for how the second ratio was determined ("÷ 16") and uses the multiplicative structure of proportional relationships to find the second ratio (MP7).

Total Awarded Points: 7 out of 8

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Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

a. Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.

Kara's = 144 Square units 12×12=144 12×12=122

b. Write two different but equivalent ratios that compare the area of the larger square to the area of the smaller square. Explain how you found both ratios.



Guide 2	Litho 6021
Total Content Points: 3	(6 RP.1, 6.RP.3, 6.EE.1)
Total Practice Points: 3	(MP1, MP2, MP4)

In this response the student uses ratios to show the relationship between the areas of the two squares ("36:4; 9:1") (6.RP.1). The student uses ratios and/or rate reasoning to find two equivalent ratios and writes an equation containing an exponent to represent the area of Kara's square (" $12 \times 12 = 12^{2^{\circ}}$ ) (6.RP.3, 6.EE.1). All parts of the problem are complete, and the student recognizes how to determine the area of the enlarged square and how to express the ratio of the areas (MP1). The student understands 3 times longer and 3 times wider and labels quantities (MP2). Ratios and expressions are used to model Kara's new square (MP4). The explanation in Part B is vague, and thus not precise (no credit for MP6). There is no explanation provided for how the second ratio is determined, and the multiplicative structure of proportional relationships to find the second ratio is not shown (no credit for MP7).

Total Awarded Points: 6 out of 8

Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

a. Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.





Guide 3	Litho 6147
Total Content Points: 3	(6.RP.1, 6.RP.3, 6.EE.1)
Total Practice Points: 3	(MP1, MP2, MP4)

In this response the student uses ratios to show the relationship between the areas of the two squares ("144:16 or 9:1") (6.RP.1). The student uses ratio and/or rate reasoning to find two different equivalent ratios (6.RP.3), and writes and evaluates one expression containing an exponent to represent the area of Kara's square (" $12^2 = 144$ ") (6.EE.1). All parts of the problem are complete, and the student recognizes how to determine the area of the enlarged square in Part A and how to express the ratio of areas in Part B (MP1). The response shows an understanding of 3 times longer and 3 times wider (" $4 \times 3$  equals 12") and includes an expression scaling both the length and width of the square (" $12 \times 12 = 144$ ;  $4 \times 4 = 16$ ;  $4 \times 3 = 12$ ") (MP2). The student uses equations to model Kara's new square (MP4), but there is no labeling of numerical quantities (no credit for MP6). There is no explanation for how the second ratio is determined (no credit for MP7).

Total Awarded Points: 6 out of 8

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#### Task 3. Comparing Squares Task

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b.

Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.

2X12 = 122 square units Kara's square = 16 square units 122=144 square units

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Guide 4	Litho 6175	
Total Content Points: 2	(6.RP.1, 6.EE.1)	
Total Practice Points: 3	(MP1, MP2, MP4)	

In this response the student uses ratios and/or ratio language to show the relationship between the area of the two squares ("144 to 16") (6.RP.1), but an equivalent ratio is not found ("24 to 1") (no credit for 6.RP.3). The student writes expressions containing exponents to represent the area of Kara's square (" $12 \times 12 = 12^2$  square units;  $12^2 = 144$  square units") (6.EE.1). All parts of the problem are complete, as the student recognizes how to determine the area of the enlarged square and how to express the ratio of the areas (MP1). The student understands the 3 times longer and 3 times wider,  $4^2$  and  $12^2$ , writes an equation scaling both the length and width of the square, contextualizes the results as the area of the larger square, and labels quantities (MP2). Equations are used to model Kara's new square (MP4). However, the response is not precise

(" $\frac{24}{1}$ ") and does not provide an adequate explanation for how the second ratio is determined

(no credit for MP6, no credit for MP7).

Total Awarded Points: 5 out of 8

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a.

Kara knows that she can find the area of a square that is <u>4 units</u> on <u>all sides</u> by using the expression  $4 \times 4$ . She also knows that this is <u>equivalent</u> to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.





Guide 5	Litho 6189	
Total Content Points: 2	(6.RP.1, 6.EE.1)	
Total Practice Points: 2	(MP2, MP4)	

In this response the student uses ratios to show the relationship to the two areas ("144:16") (6.RP.1). The student does not use ratios or rate reasoning to find an equivalent ratio (no credit for 6.RP.3). There is an expression containing exponents representing Kara's new square ("12<sup>2</sup> sq") (6.EE.1). The response lacks a second ratio and therefore does not complete all parts of the task (no credit for MP1). There is understanding of the 3 times longer and 3 times wider, and both lengths and widths of the squares are shown (MP2). The student models Kara's new square (MP4). There are no expressions or equations using exponents accurately (no credit for MP6). The response does not use the multiplicative structure of proportional relations to find the second ratio (no credit for MP7).

Total Awarded Points: 4 out of 8

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# Task 3. Comparing Squares Task

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b.

Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

a. Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.



Write two different but equivalent ratios that compare the area of the larger square to the area of the smaller square. Explain how you found both ratios.



Litho#: 6183

Guide 6Litho 6183Total Content Points: 1(6.EE.1)Total Practice Points: 2(MP2, MP4)

In this response the student does not use ratios or ratio language to show the relationship between the two areas or to find an equivalent ratio (no credit for 6.RP.1, no credit for 6.RP.3). The student writes expressions containing exponents to represent the area of Kara's square ("12 x  $12 = 12^2$  square units") (6.EE.1). The student does not complete all parts of the problem, failing to express the ratio of the areas (no credit for MP1). The student understands 3 times longer and 3 times wider ("4 x 4; 12 x 12") and finds 144 square units (MP2). Equations are used to model Kara's new square (MP4). Part B lacks an adequate answer, which demonstrates a lack of precision (no credit for MP6). There is no explanation for how the second ratio is determined (no credit for MP7).

Total Awarded Points: 3 out of 8

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b. :

Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.



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Litho#: 6129				Page 18

Guide 7	Litho 6129
Total Content Points: 1	(6.EE.1)
Total Practice Points: 2	(MP2, MP4)

In this response the student does not use ratios and/or ratio language to show the relationship between the two areas and does not find an equivalent ratio (no credit for 6.RP.1, no credit for 6.RP.3). The student writes and evaluates expressions containing exponents to represent the area of Kara's square (" $4 \times 4 = 4^2$ ,  $12 \times 12 = 12^{2^n}$ ) (6.EE.1). The student writes ratios comparing the side lengths of the two squares rather than the areas, so this response does not adequately complete all parts of the problem (no credit for MP1). The student shows understanding of the 3 times longer and 3 times wider by multiplying  $4 \times 3$ , appropriately writes an expression scaling both the length and width of the square, and labels quantities (MP2). The student uses equations to model Kara's new square (" $12 \times 12 = 144$  sq units") (MP4). As there are no labels in the second part of the problem, the ratios provided are not of the areas of the squares but rather the lengths, and the scaling factor is incorrect, the response lacks precision (no credit for MP6). There is no explanation for how the second ratio is determined and no use of the multiplicative structure of proportional relationships to find the second ratio (no credit for MP7).

Total Points Awarded: 3 out of 8

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Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

a. Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.

 $4^{2} \times 3^{2} = 144$ 

yx4×3x3 =144 Page 10 GO ON TO THE NEXT PAGE. Litho#: 6213 Page 20

Guide 8	Litho 6213
Total Content Points: 1	(6.EE.1)
Total Practice Points: 1	(MP4)

In this response the student does not use ratios or ratio language to show the relationship between the areas of the two squares and does not find an equivalent ratio (no credit for 6.RP.1, no credit for 6.RP.3). The student writes and evaluates the expression (" $4^2 \times 3^2 = 144$ ") (6.EE.1). By not providing an equivalent ratio, the student does not complete all parts of the problem (no credit for MP1). The student does not appropriately write an expression scaling both the length and width of the square, contextualize the results as the area of the larger square, or label quantities (no credit for MP2). The student uses an equation to model Kara's new square (" $4^2 \times 3^2 = 144$ ") (MP4). There are no labels and the response lacks a carefully formatted explanation (no credit for MP6), and there is no explanation for how the second ratio is determined and no use of the multiplicative structure of ratios (no credit for MP7).

Total Awarded Points: 2 out of 8

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b.

Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.

 $3 \times 3 = 3^{2}$   $4 \times 4 = 4^{2}$ 

12 4 3 9	$\frac{13}{3} = \frac{1}{3}$
i multiplied	
Page 10	GO ON TO THE NEXT PAGE.

Guide 9 Litho 6219

Total Content Points: 0

Total Practice Points: 0

In this response the student does not use ratios or ratio language to show the correct relationship between the two areas, and does not find an equivalent ratio for the area (no credit for 6.RP.1, no credit for 6.RP.3). Although there is an expression with exponents that could represent Kara's square ( $12 \times 12 = 12^2$ ), the inclusion of an incorrect expression ( $3 \times 3 = 3^2$ ) and the lack of evaluating any of the expressions do not signify clear understanding as to how area should be represented, and thus the student receives no credit for representing Kara's square or for correctly creating expressions containing exponents (no credit for MP4, no credit for 6.EE.1). The student does not recognize how to determine the area of the enlarged square and how to express the ratio of the areas, and does not label any units (no credit for MP1, no credit for MP2). The response lacks a carefully formulated explanation, and the explanation for how the second ratio is determined is vague and nonspecific (no credit for MP6, no credit for MP7).

Total Awarded Points: 0 out of 8

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Kara knows that she can find the area of a square that is 4 units on all sides by using the expression  $4 \times 4$ . She also knows that this is equivalent to writing  $4^2$ .



For a class assignment, Kara must construct a square that is 3 times longer and 3 times wider than the square above.

a. Write two different expressions to represent the area of Kara's new square. One of them must contain exponents. Use Kara's expressions and your expressions to find the areas of both squares.



b. Write two different but equivalent ratios that compare the area of the larger square to the area of the smaller square. Explain how you found both ratios.



Litho#: 6187

Guide 10 Litho 6187

Total Content Points: 0

Total Practice Points: 0

In this response the student does not use ratio or ratio language to show the relationship between the two areas and does not find an equivalent ratio (no credit for 6.RP.1, no credit for 6.RP.3). There are no expressions containing exponents to represent the area of Kara's square (no credit for 6.EE.1). The student does not recognize how to determine the area of the

enlarged square ("7 x 7 or 7<sup>2</sup>"), recognize how to express the ratio of the areas (" $\frac{4^2}{16^2 \text{ sq.}}$ "), or

complete all parts of the problem (no credit for MP1). There is no demonstration of an understanding of 3 times longer and 3 times wider ("add the 3 units longer and wider") (no credit for MP2). There is no model of Kara's new square (no credit for MP4). The student does not accurately complete the task and does not provide an explanation for how the second ratio is determined (no credit for MP6, no credit for MP7).

Total Awarded Points: 0 out of 8