Tennessee Comprehensive Assessment Program / Mathematics

TCAP/CRA 2012-2013



Task 1: Broken Light Bulbs Task Full Scoring Guide

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Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

Total Number of Light Bulbs in Case	Number of Broken Light Bulbs
150	3
250	5
600	12

- a. Prove that the data in Tamela's table indicate a proportional relationship.
- b. Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

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1. Broken Light Bulbs Task Scoring Guide

The CCSS for Mathematical Content (2 points)

- 7.RP.2a The student explains how to use values in the table to test for proportionality in any of the following ways:
 - by dividing: (*total bulbs divided by broken bulbs*) $\frac{150}{3}$, $\frac{250}{5}$, and $\frac{600}{12}$; OR (*broken bulbs by total bulbs*) $\frac{3}{150}$, $\frac{5}{250}$, and $\frac{12}{600}$; observing that the quotient is the same for all three ratios.
 - by representing the ratios in equivalent fraction form to the unit rate, e.g., $\frac{150 \text{ bulbs}}{3 \text{ broken bulbs}} = \frac{250 \text{ bulbs}}{5 \text{ broken bulbs}} = \frac{600 \text{ bulbs}}{12 \text{ broken bulbs}} = \frac{50 \text{ bulbs}}{1 \text{ broken bulbs}} \text{ OR}$ $\frac{3 \text{ broken bulbs}}{150 \text{ bulbs}} = \frac{5 \text{ broken bulbs}}{250 \text{ bulbs}} = \frac{12 \text{ broken bulbs}}{600 \text{ bulbs}} = \frac{1 \text{ broken bulb}}{50 \text{ bulbs}}.$
 - by scaling up the unit rate in fraction form, e.g., $\frac{50 \text{ bulbs}}{1 \text{ broken bulb}} = \frac{150 \text{ bulbs}}{3 \text{ broken bulbs}} = \frac{250 \text{ bulbs}}{5 \text{ broken bulbs}} = \frac{600 \text{ bulbs}}{12 \text{ broken bulbs}}.$
 - by testing in fraction form to see if $\frac{150 \text{ bulbs}}{3 \text{ broken bulbs}} = \frac{250 \text{ bulbs}}{5 \text{ broken bulbs}} = \frac{600 \text{ bulbs}}{12 \text{ broken bulbs}} \text{ scaling up, e.g.,}$ $\frac{150 \times 4}{3 \times 4} = \frac{600}{12}.$
 - by showing that scale factor is the same for each line of the table.
- 7.RP.3 The student finds the number of broken bulbs by applying the percent of broken bulbs to a case of 800 bulbs. The student may do this in one of the following ways:
 - by dividing and multiplying, e.g., $5 \div 250 = .02$ and $.02 = \frac{2}{100} = 2\%$; $.02 \times 800 = 16$.
 - by using ratios or proportions, e.g.; $\frac{3}{150}$ or $\frac{x}{100}$ to determine 2%; $\frac{x \text{ broken bulbs}}{800 \text{ bulbs}} = \frac{2 \text{ broken bulbs}}{100 \text{ bulbs}}$, and solve for x or scale to $\frac{16}{800}$.
 - by scaling the unit rate in fraction form, e.g., $\frac{1}{50} = \frac{?}{100}$ to determine 2%;

 $\frac{2}{100} = \frac{?}{800}$ and solve for ?

The CCSS for Mathematical Practice (4 points)

MP1 The student proves that all the entries in the table meet the conditions of a proportional relationship. Responds to all parts of the task.

(MP1: Make sense of problems and persevere in solving them.)

MP2 The student uses the data from the table, forms ratios, equations, or graphs to test for proportionality; provides labels or explanations in Part B referencing the context of the situation.

(MP2: Reason abstractly and quantitatively.)

MP6 The student proves the data in the table indicate a proportional relationship; determines the percent of broken light bulbs per case, predicts the number of broken light bulbs in a case of 800 bulbs and provides accurate work or explanation.

(MP6: Attend to precision.)

MP7 The student demonstrates understanding of the structure of proportional relationships using equations, the constant of proportionality (unit rate), or scaling.

(MP7: Look for and make use of structure.)

Total Practice Points _____

Total Awarded Points _____

The CCSS for Mathematical Content Addressed in This Task

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Recognize and represent proportional relationships between quantities.

- 7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

The CCSS for Mathematical Practice*

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

*Gray text indicates Mathematical Practices that are not addressed in this task.

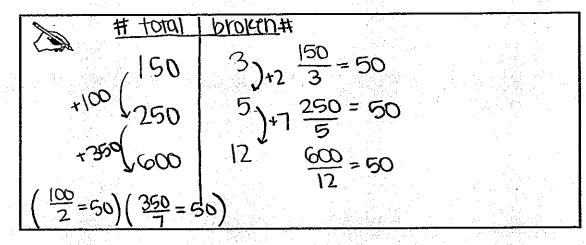
Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

b.

Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

Total4Numberoi. MantBulbsunCase	NumberofBolen LughtBulles
150	3
250	5
600	12

a. Prove that the data in Tamela's table indicate a proportional relationship.



Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

Case 1 = 3-150 = .02 = 2°1. broken Case 2=5:250=.02=2:1. broken (ase 3 = 17: -600 = .02 = 2°1. broken case of $\frac{01}{800} = 16 \div 800 = .02 = 2°10 broken$

GO ON TO THE NEXT PAGE.

Page 6

Litho#: 7081

Guide 1	Litho 7081
Total Content Points: 2	(7.RP.2a, 7.RP.3)
Total Practice Points: 4	(MP1, MP2, MP6, MP7)

The student tests for proportionality by using division to determine a unit rate of 50 for all three entries (7.RP.2a). The student divides the number of broken bulbs by the number of total bulbs to calculate that 2% of the bulbs are broken in each case, and then uses the same process to predict that there would be 16 broken bulbs in a case of 800 (7.RP.3). The work demonstrates understanding of the structure of proportional relationships by using ratios to determine unit rates (MP7). The student provides correct work in Part A and labels in Part B, demonstrating correct reasoning and relation to the context of the task (MP2). The student provides accurate work proving that the data in the table represent a proportional relationship, predicting 16 broken bulbs in a case of 800 and finding the percentage of broken bulbs in each case (MP6), making sense of and persevering in solving all parts of the task (MP1).

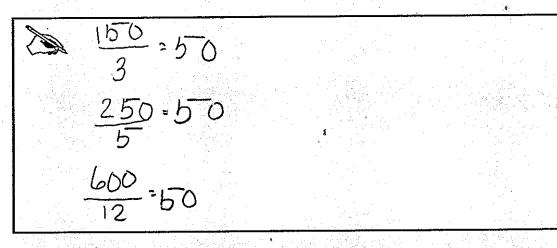
Total Awarded Points: 6 out of 6

b.

Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

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150	3
250	5
600	12

a. Prove that the data in Tamela's table indicate a proportional relationship.



Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

 $\frac{800}{16} = 50$ Page 6 GO ON TO THE NEXT PAGE.

Litho#: 7091

Guide 2 Litho 7091

Total Content Points: 2 (7.RP.2a, 7.RP.3)

Total Practice Points: 4 (MP1, MP2, MP6, MP7)

The student tests for proportionality by using division to determine a unit rate of 50 for all three entries (7.RP.2a). The student determines that 2% of the bulbs are broken in each case and multiplies the percent by 800 bulbs to predict that there will be 16 bulbs in a case of 800 (7.RP.3). The student demonstrates understanding of the structure of proportional relationships by using ratios to find the constant of proportionality from the table data (MP7) and provides sufficient labeling in Part B and work in Part A to show clear reasoning relevant to the context of the situation (MP2). The work is correct and sufficient to prove a proportional relationship exists (MP6). The student has correctly and completely addressed all parts of the task (MP1).

Total Awarded Points: 6 out of 6

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b.

Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

Guide 3

Total/Number of the Light/Bulbs in Case	Number of Broken Light Bulbs
150	3
250	5
600 ····	12

a. Prove that the data in Tamela's table indicate a proportional relationship.

It is a proportional relationship because If you take the number of light-builds in case and divide it by the number of broken lightbuilds you get 50 for each case. That is what make the table a proportional relationship.

Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

150 Light bulbs in a case with 3 broken is 2% that is broken in that Case. 200 Light bulbs in a case with 5 broken light bulbs is 2% also. 600 Light bulbs in a case with 12 broken light bulbs is also 2%. Ithink that a case of 800 light bulbs 14 of them will be broken because that is 2%. Page 6 GO ON TO THE NEXT PAGE.

Page 11

Guide 3	Litho 7055
Total Content Points: 2	(7.RP.2a, 7.RP.3)
Total Practice Points: 3	(MP1, MP2, MP7)

The student describes a test for proportionality using division to determine a unit rate of 50 that is equivalent for all three entries (7.RP.2a). The student recognizes that the percentage of broken bulbs in all of the cases is 2%, and determines that in a case of 800 bulbs, 16 will be broken (7.RP.3). The student correctly represents the table data in terms of ratios, demonstrating understanding of the structure of proportional relationships (MP7). Correct labeling for all values in part b indicates understanding of

the context of the problem (MP2). Although the equation $\frac{150}{50} = \frac{50}{B}$ shown in Part A is unclear (no credit for MP6), the work and explanations sufficiently prove the student's

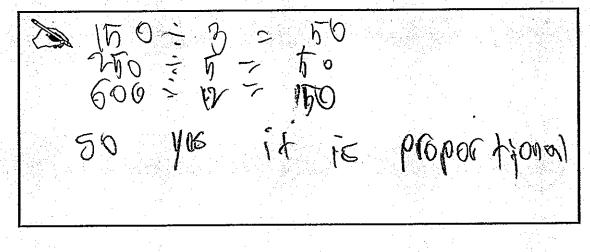
ability to make sense of the problem and persevere in solving all parts of the task (MP1).

Total Awarded Points: 5 out of 6

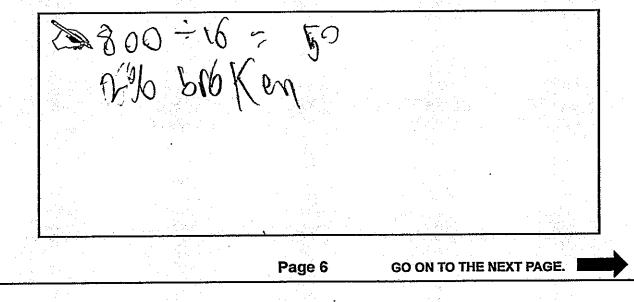
Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

TOEHAUMDERON LigntBulbsinGese	NumberonBioken. LucitaBollos
150	3
250	5
600	12

a. Prove that the data in Tamela's table indicate a proportional relationship.



b. Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.



Guide 4	Litho 7077
Total Content Points: 2	(7.RP.2a, 7.RP.3)

Total Practice Points: 2 (MP1, MP7)

The student tests for proportionality by determining that the unit rate of 50 for all three entries (7.RP.2a), demonstrating understanding of the structure of proportional relationships by using ratios (MP7). The student determines 2% of the bulbs are broken and applies this value to the case of 800 (7.RP.3). However, the lack of labeling in the equation in Part B ($800 \div 16 = 50$) makes it unclear whether 16 or 50 broken bulbs would be found in a case of 800 (no credit for MP2). This lack of clarity indicates insufficient attention to precision (no credit for MP6). The student makes sense of the problem and perseveres in attempting all parts of the task (MP1).

Total Awarded Points: 4 out of 6

a.

b.

Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

Guide 5

Page 15

liotal/Number of a Light Bulbs in Case	Number/of/Broken
150	3
250	5
600	12

Prove that the data in Tamela's table indicate a proportional relationship.

150 + 3 = 50

 $250 \div 5 = 50$ $600 \div 12 = 50$ bulls broken %

Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

800 ÷ 5 bulks 9	50=16 6 broken bulbs	
	bulbs	
	Page 6 GO ON TO	THE NEXT PAGE.
Litho#: 7115		

Guide 5	Litho 7115
Total Content Points: 2	(7.RP.2a, 7.RP.3)
Total Practice Points: 1	(MP7)

The student uses division to determine that the data in the table represent a proportional relationship, indicating that dividing the total number of bulbs by the number of broken bulbs results in the same quotient in each case (7.RP.2a). Although the student does not correctly label the result of the division, the process shown is sufficient to test for proportionality. The student uses the equation $800 \div 50 = 16$ to correctly predict the number of broken bulbs in a case of 800 (7.RP.3). The student demonstrates a conceptual misunderstanding of percentage when labeling in parts A and B (no credit for MP2). The incorrect use of percentage evidences a lack of precision (no credit for MP6), and incomplete comprehension of the task (no credit for MP1). Despite this misunderstanding of percentages, the student does indicate an ability to use the structure of proportional relationships (MP7).

Total Awarded Points: 3 out of 6

b.

Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

ເບັດສານໃນແຫຼນອາດີ. ໃນທຸດສອນໄທຣາດ Gase	NumberorBiolen LightBilbs
150	3
250	5
600	12

a. Prove that the data in Tamela's table indicate a proportional relationship.

150=50 20-0.03 2120-0.03 $\frac{D}{600} = 0.02$

Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

for all 350 4550 time from 250 i 6.0% ner the broken of 50 with 201 or would be broken assue ris 15010 GO ON TO THE NEXT PAGE. Page 6

Litho#: 7097

Guide 6

Guide 6	Litho 7097
Total Content Points: 1	(7.RP.2a)
Total Practice Points: 1	(MP7)

The student tests for proportionality by determining that the ratio of broken bulbs to total bulbs is equivalent for all three entries in the table (7.RP.2a), thereby demonstrating understanding of the structure of proportional relationships by using ratios (MP7). The student attempts to use the unit rate to predict the number of broken bulbs in a case of 800, but the answer given is incorrect (no credit for 7.RP.3). The answer in Part B is mistakenly labeled as a percentage of broken bulbs, 15%, instead of as the number of broken bulbs (no credit for MP2). The incorrect and unclear equations in Part A and the incorrect process in Part B demonstrate a lack of precision (no credit for MP6). Inverted ratios in the second column of Part A and flawed logic in Part B show that the student has not adequately made sense of the problem to complete the task (no credit for MP1).

Total Awarded Points: 2 out of 6

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a.

b.

Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

Total/Number of au Light Bulbs in Case	Number of Broken
150	3
250	5
600	12

Prove that the data in Tamela's table indicate a proportional relationship.

If you the Unit rate of the total number of light bulbs and Broken light bulbs it is proportional. 150:3= 50, t 250:3=50; it total Broken rate bulbs rate (Oth bulbs

Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

150 Total = 0,02%. Broken per Case. bulbs Bulbs 16 = 800= 0.02 Broken total broken bulbs pur °lo XX bulbs Broken unit Broken bulbs rate Per bulbs Case Page 6 GO ON TO THE NEXT PAGE:

Guide 7

Guide 7	Litho 7117
Total Content Points: 1	(7.RP.3)
Total Practice Points: 1	(MP7)

Although the student finds a unit rate of 50 using the table data, demonstrating understanding of the structure of proportional relationships (MP7), only two of the entries in the table are tested for proportionality (no credit for 7.RP.2a). The student uses the data to form ratios in order to find equivalence, but the decimal value is not converted to a percentage of broken bulbs in Part B (no credit for MP2). Despite the correct prediction of 16 broken bulbs in a case of 800 (7.RP.3), the incorrect expression of the percentage of broken bulbs and the inappropriate equation ($250 \div 3$) in Part A indicate a lack of precision (no credit for MP6). Because of the incomplete testing of values in the table and the incorrect expression of percentage of broken bulbs in each case, the student has not completed all parts of the task (no credit for MP1).

Total Awarded Points: 2 out of 6

b.

Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

Total Number of a light Bulbs in Case	Number of Brokens Eight Bulbs
150	3
250	5
600	12

a. Prove that the data in Tamela's table indicate a proportional relationship.

Each time they are	dividina
Eaut	· · · · · ·)
by 50. 150 ÷ 3= 50	
200 - 3= 50	
400 = 3-50	

Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

· · · · · · · · · · · · · · · · · · ·			Constant and the second		1.1.1
	150 light bulbs - 3 broken	0,02	% x 800	light bulbs	
	0.02%	* *			
		110	broken a	ones	
		would	ba in	800	
		light	bulbs		
		Page 6	GO ON TO TI	IE NEXT PAGE.	
		<u>_</u>			
	Litho#: 7039				

Guide 8

Guide 8 Litho 7039

Total Content Points: 1 (7.RP.3)

Total Practice Points: 0

The student attempts to show the proportionality of the data in the table by using division to show the constant of proportionality, but only the first equation is correct and the explanation is unclear (no credit for 7.RP.2a). The student correctly predicts that 16 broken bulbs would be broken in a case of 800, correctly multiplying the total number of bulbs by 0.02 (7.RP.3). The student's division of all the amounts of total bulbs by 3 demonstrates insufficient understanding of the structure of proportional relationships needed to reason through and solve the problem correctly (no credit for MP2, no credit for MP7). The response lacks precision and accuracy in the calculations and in the labeling of the percentage of broken bulbs (no credit for MP6). The incorrect approach to prove proportionality is evidence that the student has not completely made sense of the problem (no credit for MP1).

Total Awarded Points: 1 out of 6

a.

b.

Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

TOTELANDIMOEROD LATITABULDSIDCERSE	Numberoi/Bioken Lught(Bulbs
150	3
250	5
600	12

Prove that the data in Tamela's table indicate a proportional relationship.

The Data is not proportional because the Jata is not constant. It does not have a constant ROAE OF CHANGE LIKE MS. ZAZZANO tought me.

Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

 $\frac{3}{150} = 0.02 = 2\% \quad 0.0$ $\frac{3}{150} = 0.02 = 2\% \quad 0.0$ $\frac{5}{150} = 0.02 = 2\% \quad 0.0$ 0.02 = 2% Page 6 GO ON TO THE NEXT PAGE.

Litho#: 7063

Guide 9 Litho 7063

Total Content Points: 1 (7.RP.3)

Total Practice Points: 0

The student correctly calculates the percentage of broken bulbs in each case to predict that 16 bulbs would be broken in a case of 800 (7.RP.3). However, the value 16 is not clearly labeled as the number of broken bulbs, and Part A gives no evidence of correct reasoning to show proportionality (no credit for MP2). The student does not use the data from the table to show that the relationships are proportional (no credit for 7.RP.2a). The incorrect determination that entries are "not proportional" indicates a lack of understanding of the structure of proportional relationships (no credit for MP7). Although the student shows accurate and appropriate work in predicting the number of broken bulbs in a case of 800, the explanation in Part A is incorrect (no credit for MP6). The response to Part A shows that the student has not made sense of and persevered in solving all parts of the task (no credit for MP1).

Total Awarded Points: 1 out of 6

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Tamela works for I.D.E.A. Light Bulb Company. She inspects cases of light bulbs and counts the number of broken bulbs. Last week, she recorded her count in the table below.

Guide 10

	Total Number of	e ie	Number of Broken Number of Broken Light Bulbs
л ¹ 1	150	•	3
•	250		5
	600		12

a. Prove that the data in Tamela's table indicate a proportional relationship.

go in to the numbers.

b. Determine the percent of broken bulbs in each case and use that percentage to predict the number of broken light bulbs in a case of 800 bulbs. Use mathematical reasoning to justify your response.

800 bulbs in the 16 out soo are broken Page 6 GO ON TO THE NEXT PAGE. Litho#: 7029 Page 25

Guide 10

Litho 7029

Total Content Points: 0

Total Practice Points: 0

The student does not clearly use the data in the table to test for proportionality (no credit for MP7). Although the student indicates use of division to determine proportionality, the explanation provided is imprecise and unclear (no credit for 7.RP.2a, no credit for MP6). The student does not show work, explain a process for calculating a percentage, or otherwise prove how the correct answer of 16 bulbs was found (no credit for 7.RP.3). The student has not responded to all parts of the task, as the explanation in Part A is insufficient, there is no attempt to calculate the percentage of broken bulbs in each case, and there is no explanation for the answer in Part B (no credit for MP1). Although there is correct labeling in Part B, the student does not clearly show or explain the reasoning used to determine proportionality in Part A (no credit for MP2).

Total Awarded Points: 0 out of 6