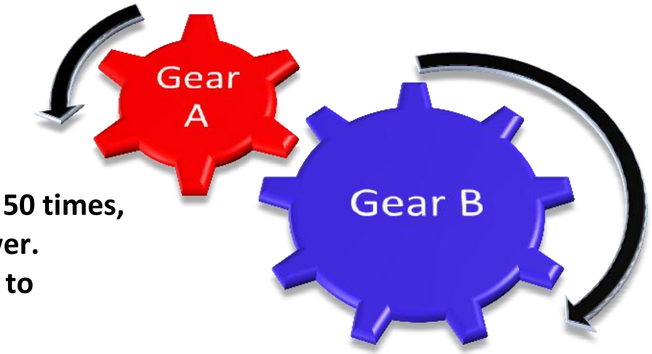
Task: Gears Task	6th Grade
Jason is working to fix his grandmother's clock and finds that the gears are not aligned and not working properly. He becomes curious about how the gears work. **A. If gear A turns three times, will gear B turn more times, fewer times, or the same amount? Explain your reasoning.** **B. Find a way to record the relationship between turns for each gear as they turn together several times. Describe any patterns you see.** **C. If gear B turns one time, how many times will gear A turn? If gear B turns 50 times, How many times will gear A turn? Explain how you determined your answer.** **D. If 4 teeth are added to gear A, how many teeth should be added to gear B to maintain the gear ratio? Explain how you determined your answer.**	



Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* 2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* 3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- Reasoning with ratios involves attending to and coordinating two quantities.
- A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
- A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the

<table>
<tr><td colspan="2">corresponding values of the quantities change.</td></tr>
<tr><td colspan="2">Explore Phase</td></tr>
<tr><td>Possible Solution Paths</td><td>Assessing and Advancing Questions</td></tr>
<tr><td><u>PART A:</u>
As gear A turns, gear B turns fewer times.
Students may say that gear A is smaller/fewer teeth, so gear B will turn more slowly.
Students may say that as the 6 teeth from gear A align to 6 teeth of gear B, there are three more teeth for gear B and gear A is going around again, so gear B turns more slowly.</td><td><u>Assessing Questions</u>
Tell me why you think gear B will turn fewer times, or more slowly than gear A.
<u>Advancing Questions</u>
Do you think there is a relationship between the turns of the gears? How could you determine that relationship?</td></tr>
<tr><td><u>PART B:</u>
Students may record a table, horizontally or vertically:
<table>
<tr><th>Gear A turns</th><th>Gear B turns</th></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>6</td><td>4</td></tr>
<tr><td>9</td><td>6</td></tr>
<tr><td>12</td><td>8</td></tr>
<tr><td>18</td><td>12</td></tr>
<tr><td>36</td><td>24</td></tr>
</table>*Students may recognize that the teeth and turn ratios are opposite.
Students may say they notice the turns for gear A are always 1.5 times that for gear B.
Depending on how students record the values in the table, they may say that the number of turns goes up by 3 each time for gear A and two each time for gear B.
Students may say that the gear A turns are all multiples of 3 and the gear B turns are all multiples of B.</td><td><u>Assessing Questions</u>
Tell me why you decided to record the values in a table. How did you determine the values for the number of turns for each gear?
<u>Advancing Questions</u>
What patterns do you notice in your table? How is that related to the teeth on each gear? If gear A turns 10 times, how could you determine how many times gear B turns?</td></tr>
<tr><td><u>PART C:</u>
If gear B turns one time, gear A will turn 1.5 times.
Students may express the ratio as 1:1.5.
Students may divide any row on the table by the number of turns for gear B to scale it down to 1, and gear A will be scaled to 1.5.
If gear B turns 50 times, gear A will turn 75 times.
Students may scale the unit rate: 50 x 1.5 = 75.
Students may extend the table until they have a row with 75 and</td><td><u>Assessing Questions</u>
How do you know gear A will turn 1.5 times? How did you determine this value?
Explain to me how you determined 75.
<u>Advancing Questions</u>
You found that gear A will turn 1.5 times if gear B turns 1 time. How can you use this relationship to determine how many times gear A will turn if gear B turns 50 times?</td></tr>
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50. Students may set up a proportion: $\frac{2}{3}=\frac{50}{x}$, x = 75. Students may scale the 2:3 ratio by a factor of 25, so it will now be 50:75.	What is the relationship between these values and the patterns you described in part B?
<u>PART D:</u> If 4 teeth are added to gear A, 6 teeth should be added to gear B. Students may refer to the table and check that the ratio is maintained. Students may set up an equation: y = 3/2x (y is number of teeth for gear B, x is number of teeth for gear A) y = 3/2(6 + 4) = 3/2 (10) = 15. So, 6 teeth should be added to gear B. 2y = 3x (y is number of teeth for gear B, x is number of teeth for gear 1) 2y = 3(10) (10 comes from 6 plus 4) y = 15, 6 more than 9. So, 6 teeth should be added to gear B. Adding 4 more to gear A will make 10 teeth. Students may scale the unit rate by a factor of 10, 1.5 x 10 = 15. So, 6 teeth should be added to gear B.	<u>Assessing Questions</u> Tell me how you determined how many teeth should be added to gear B. What did you have to consider? <u>Advancing Questions</u> How can you be sure your value is correct? How is this value connected to the rate you found in part C and the relationships in the table you set up in part B?
Possible Student Misconceptions	
<u>PART A:</u> Students may think they turn at the same rate. Students may think that they gear with more teeth will turn at a faster rate.	<u>Assessing Questions</u> Look at the alignment of the teeth between the gears. What will happen to gear B as gear A goes all the way around? <u>Advancing Questions</u> What will happen to gear B as gear A continues to turn? What is the relationship between the gears?
<u>PART B:</u> Students may not know how to record the number of turns for each gear together. For example, if students increase gear turns for gear A in single number increments, they may not be able to also consider how gear B will change simultaneously.	<u>Assessing Questions</u> How can you organize values for the number of turns for gear A and gear B? What do you need to consider? <u>Advancing Questions</u> How are the values you have for gear A related? Gear B? What is the

	relationship between turns for each row you have?
<u>PART C:</u> Students may not see how to use the unit rate to scale up. Students may use the ratio incorrectly and think 33.	<u>Assessing Questions</u> Will gear A turn more or less than 1 turn if gear B turns 1 time? Why? What if gear B turns 2 times? <u>Advancing Questions</u> What will happen to gear A as gear B continues to turn? Can you use that relationship to find out what will happen if gear B turns 50 times?
<u>PART D:</u> Students may think that the same amount, 4, should be added to both gears to keep the relationship the same.	<u>Assessing Questions</u> Tell me why you think you should add 4 to both gears to maintain the ratio. What is the ratio you are trying to maintain? <u>Advancing Questions</u> Look at your table of values and try to add 4 to both gears. What happens?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	<u>Assessing Questions</u> Let's look the gears. As gear A turns, tell me what will happen to gear B. Why? <u>Advancing Questions</u> If gear A turns all the way around, will gear B also turn all the way around? Why or why not?
If students finish early….	<u>Assessing Questions</u> Show me how you determined your answer to part D. <u>Advancing Questions</u> If gear B turns 117 times, how many times will gear 1 turn? How do you know? If a third gear C was added and connected to gear B with 15 teeth and gear A turned 5 times, how many times would gear C turn?
Discuss/Analyze	
Whole Group Questions	
<u>PART A:</u> Who can explain how you know that gear B will turn fewer times than gear A? Does it matter how many times gear A turns – will gear B always turn fewer times? Why?	
<u>PART B:</u> Many of you created a table to record the turn values. Why was this an effective way to keep track of the turns? What are some of the patterns that you noticed? How did this table help you establish a relationship between the number of turns? What is the relationship between gear turns? Did anyone find another way to keep track of the gear turns? Was it more useful than a table?	

PART C: Who can explain how you determined the number of times gear A would turn if gear B turned once? Who used the table? How did that help? Who set up an equation? How did you determine what equation to use? Who used a ratio and scaled through division or multiplication? How are these methods related to each other?
PART D: Who initially thought you would add 4 teeth to both gears? Why? What made you change your mind? If 4 teeth are added to gear A and 6 teeth are added to gear B, how does that connect to the relationship you saw in part B?