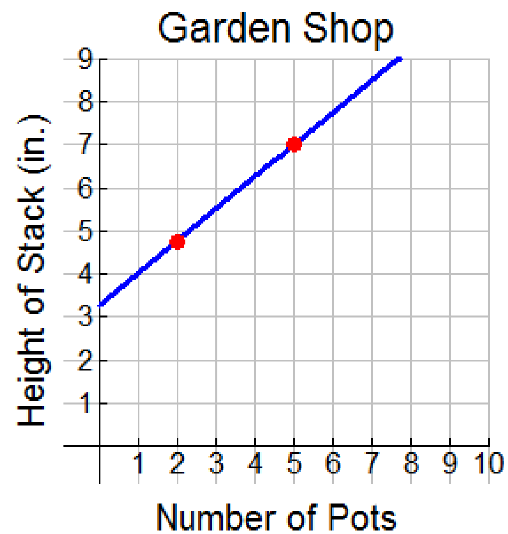
Task: Garden Center

Elizabeth and Juno stock shelves at the garden center. The garden center receives a shipment of flower pots. Each girl makes a guess as to how many containers of the same size and shape can be stacked on a shelf. The pot that they selected is pictured below. Consider the linear graph they drew to model the heights of the pots that includes two points they found by measuring a number of pots and the height of the stack that they form -- $\left(2, 4\frac{3}{4}\right)$ *and* $(5, 7)$.



a. Describe in your own words the functional relationship between the two quantities illustrated by the graph.
b. Write an equation to calculate the height of the stack for any number of pots.
c. If there is 3.5 feet of vertical space on, calculate how many pots can be stored in one stack?
d. How would the graph and the equation change if the pot shape was 10 inches tall with a 1. 5 inch lip?

Teacher Notes:

If flower pots with pronounced lips (as illustrated in the picture) are not available, students could use Styrofoam coffee cups that stack in similar ways to provide a hands-on investigation of how stacking objects that fit one in to another operate. By collecting data on such a pattern of cups, students can build background knowledge so that they will be aware that having two cups does not double the height of the stack and consequently a direct variation is not an appropriate model for this pattern.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (X,Y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- Functions provide a tool for describing how variables change together. Using a function in this way is called *modeling*, and the function is called a *model*.
- Functions can be represented in multiple ways—in algebraic symbols, situations, graphs, verbal descriptions, tables, and so on—and these representations, and the links among them, are useful in analyzing patterns of change.
- Some representations of a function may be more useful than others, depending on how they are used.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions
Part (a) The functional relationship between the height of the stack in inches and the number of pots in the graph appears to be a linear relationship with a positive slope. As the number of pots increases then the height of the stack of pots increases. Although the model drawn in the first quadrant to represent this relationship appears as a continuous straight line, for each pot added the stack increases in height an additional ¾ inches making the functional relation a set of ordered pairs where the domain is the set of positive integers.	<u>Assessing Questions</u> • What is being measured on the horizontal axis? (How is it changing?) • What is being measured on the vertical axis? (How is it changing?) <u>Advancing Questions</u> • What is the meaning of the y-intercept in this context? • Explain the meaning of the unit rate of change for these quantities in the context of the problem situation.
Part (b) Depending on their background knowledge students may create the equation in one of several ways. (1) Students may use the shape of the pot to generate ordered pairs like (1,4), (2, 4 ¾), etc. based on information from the picture. Using these points in the slope formula, they can find that the slope is ¾ and by substituting a point into the	<u>Assessing Questions</u> • Which form of the line will help to create an equation for the graph? Why did you select this form? • How can you use two points to find the slope of a line? • Explain how you found the y-intercept. • Does the value that you found when you calculated the y-

equation y = mx + b, the value of b can be computed as 3 ¼ to give the equation f(x) = ¾ x + 3 ¼ in slope-intercept form where f(x) is the height of the stack in inches for x number of pots. (2) After considering the picture of the pot, students may consider the linear graph as a set of ordered pairs representing an arithmetic sequence where the number of pots indicates the term number and the height of the stack of pots gives the value of each term. By using the formula for the a_n term of an arithmetic sequence $\{a_n = a_1 + d(n-1)\}$ students may generate the equation y = 4 + ¾ (x-1) which is equivalent to f(x) = ¾ x + 3 ¼ . (3) Since two points are given in the problem statement, students may use the two points to generate the slope and then the point-slope form of the line (y – 7) = ¾ (x – 5) or (y – 4 ¾) = ¾ (x – 2) to find the slope-intercept form of the line f(x) = ¾ x + 3 ¼ .	intercept seem to agree with the graph? Why do you think so? <u>Advancing Questions</u> • Explain how the lip of the flower pot relates to the linear equation. • Explain how the measured height of the pot relates to the y-intercept of your linear equation. • Sam claims "Two pots will have a height of 8 inches." Explain why this result does not agree with the answer produced by the linear model when x = 2.
Part © Given the shelves are 3.5 feet apart, students will need to convert that to inches [3.5 ft x (12 in/1 ft)] to find that the shelves are 42 inches apart. Using 42 inches as the y or f(x) value for the linear function, students can solve the equation 42 = ¾ x + 3 ¼ for x to find x = 51 2/3 pots. In the context of the problem, this means that 51 stacked pots can fit on the shelves.	<u>Assessing Questions</u> • With the dimensions of the pot given in inches and the width between shelves given as 3.5 feet, what aspect of precision should be attended to here as students work to find the number of pots that will fit on the shelf? • How do you go about changing 3.5 feet to inches? <u>Advancing Questions</u> • Marcia claims that 51 stacked pots will fit on the shelf. Explain why you agree or disagree with Marcia. • How many pots would fit on the shelves stacked vertically if the distance between shelves was one meter?
Part (d) The linear model for this function would have a y-intercept of 8.5 inches and a slope of 1.5 inches/pot. The graph of the linear model would start higher on the y-axis and have a steeper slope. As mentioned in Part (a) above, the graphical model of the relationship showing the number of pots and the height of the stack in inches appears as a line in the first quadrant. It is actually comprised of a set discrete ordered pairs where the x-value is a	<u>Assessing Questions</u> • How will the rate of 1.5 inches / pot impact the equation for the relationship between the number of pots and the height of the stack of pots stacked vertically? • How does the y-intercept of the linear model for this function relate to the 10-inch height of one pot?

positive integer and the y-value is a positive rational number.	<u>Advancing Questions</u> • What is the height of 2 ½ pots? • What is the height of a stack of zero pots? How does that relate to the y-intercept of the linear model?
Possible Student Misconceptions	
A student may treat a data point as if it belongs to a proportional relationship. *Example*: Since the data point (5,7) indicates that 5 pots create a stack that is 7 inches tall, then a stack of 10 pots must have a height of 14 inches.	<u>Assessing Questions</u> • Based on the linear equation that you wrote in part (b), what is the height of the stack for 10 pots? Does this agree with 10 pots having a height of 14 inches? <u>Advancing Questions</u> • Based on the context of the problem, will every addition of 4 pots increase the height of the stack by 3 inches regardless of how many pots are already in the stack?
Students may mistake the y-intercept for the height of the first pot.	<u>Assessing Questions</u> • How tall is the one pot in the diagram? Does that value agree with the y-intercept of the linear model in the graph? Why or why not? <u>Advancing Questions</u> • What attributes of the pot create the y-intercept of the linear model?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	<u>Assessing Questions</u> • What is meant by the point (2, 4 ¾)? … (5,7)? • Explain the meaning of the y-intercept in terms of the context of the problem. <u>Advancing Questions</u> • Why is the y-intercept of the model not 4 inches? • Will the height of the stack of pots ever be 8 inches tall? Explain.
If students finish early….	<u>Assessing Questions</u> • Jim created a linear model based on data he found for stacking chairs. His equation was $y = 4x + 60$ where x is the number of chairs and y is the height of the stack in inches. Explain the meaning of the slope and y-intercept in the context of the problem.

	• Max looked at Jim's equation for stacking chairs and determined that the original chair must be 60 inches tall. Explain to Jim why he is mistaken. Advancing Questions • Find a stackable object in the classroom and explain how to create a model that gives the height of the stack depending on the number of objects. • As the answer to part (d) Mark wrote $y = 1.5(x-1) + 10$ where x is the number of pots and y is the height of the stack in inches. Explain whether Mark's answer is correct and include your reasoning. • How many pots will fit in a shelf that is 30 inches tall?

Discuss/Analyze

Whole Group Questions

- Explain how the dimensions of the flower pot can be used to find the y-intercept of a linear model used to find the height of the stack of pots when the number of pots is known?
- How can you use the linear model you created in part (b) to find out how many pots are in a stack that is 37 inches tall?
- What other types of objects stack in ways similar to the flower pots?
- If the model for a new stack of similar pots is $y = 2x + 9$, what can you predict about the features of the flower pot?