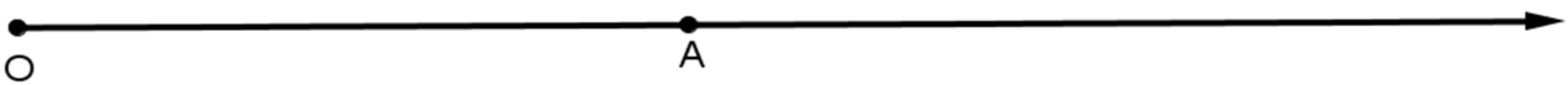
Task: Scaling Points	**5th Grade**

The number A is placed on the number line below.



a) Bennie must multiply the number A by $\frac{3}{5}$. Where would Bennie's number be placed on the number line? Why?

b) Chelsea must multiply the number A by $\frac{4}{3}$. Where would Chelsea's number be placed on the number line? Why?

c) Darnell must multiply the number A by $\frac{13}{6}$. Where would Darnell's number be placed on the number line? Why?

d) Evan must multiply the number A by $\frac{5}{8}$. Where would Evan's number be placed on the number line? Why?

e) If the number represented by A is multiplied by another factor, when will the product of A and the other factor be between 0 and A on the number line? Why?

f) If the number represented by A is multiplied by another factor, when will the product of A and the other factor be to the right of A on the number line? Why?

Teacher Notes:

Students may have difficulty working with the point A on the number line. The point A represents an arbitrary point, and there are no other points (other than the origin 0) on the number line to show the relative size of A. The idea of A being an arbitrary point is a very abstract concept, and students may need something more concrete to begin the problem.

Students may divide the distance between 0 and A into equal parts in a number of ways, including using a ruler to measure the distance between 0 and A, using paper folding, or estimating distances. Note that the point A on the number line was arbitrarily placed and the distance between 0 and A is not a whole number distance in any standard units. Depending on ability levels of students, teachers may want to re-draw the number line to ensure that the distance between 0 and A is a whole number distance in some unit and allow students to use rulers to make their measurements. Since the denominators of the fractions change with each part, if a teacher decides to re-draw the number line, she may also want to change the fractions.

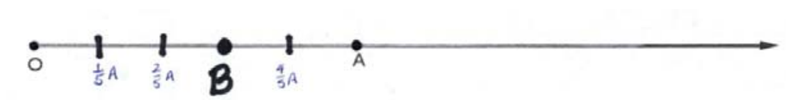
Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
5.NF.B.5 Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n* x *a*)/(*n* x *b*) to the effect of multiplying *a*/*b* by 1.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- In the multiplicative expression *A* X *B*, *A* can be defined as a *scaling factor*.
- A situation that can be represented by multiplication has an element that represents the scalar and an element that represents the quantity to which the scalar applies.
- Multiplying a positive number A by a number between 0 and 1 results in product that is smaller than A. Multiplying a positive number A by a number larger than 1 results in a product that is larger than A.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions
Part (a): To multiply the number A by $\frac{3}{5}$, students may approach the problem in a number of ways. **Approach 1:** Students may interpret $\frac{3}{5}$ times A as "$\frac{3}{5}$ of A". They would then take the distance between 0 and A on the number line,	**Assessing Questions:** How did you determine where to place Bennie's point? How did you know that Bennie's point would be between 0 and A? **Advancing Questions:** Suppose we know that the value of A is 5. How could we figure out

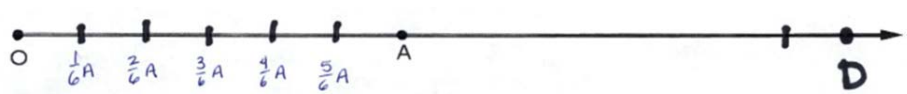
divide it into five equal spaces, then choose a point that represents $\frac{3}{5}$ of the distance between 0 and A. (Students may either use a ruler to measure as precisely as possible, use paper folding, or estimate the distances. In the latter case, they will need to make sure their fifths are close to the same size.) **Approach 2:** Students may interpret $\frac{3}{5}$ times A as "3 times A divided by 5". In this case, they would triple the distance between 0 and A, then divide the distance between 0 and 3 times A into five equal parts and choose the point representing the first part. The picture below shows the point B as representing $\frac{3}{5}$ times A using Approach 1. Approach 2 will give you the same point, but students will need to extend the number line in order to place 3 times A on the number line. 	where to place $\frac{3}{5}$ x 5 on the number line? How can we use this idea to determine where to place Bennie's point?
Part (b): To multiply the number A by $\frac{4}{3}$, students may approach the problem in a number of ways. **Approach 1:** Students may interpret $\frac{4}{3}$ times A as "$\frac{4}{3}$ of A". They would then take the distance between 0 and A on the number line, divide it into three equal spaces, then choose a point that represents $\frac{4}{3}$	**Assessing Questions:** How did you determine where to place Chelsea's point? How did you know that Chelsea's point would be to the right of A on the number line? **Advancing Questions:** Suppose we know that the value of A is 6. How could we figure out

of the distance between 0 and A. Note that students will need to extend their "thirds" past A to the right in order to choose $\frac{4}{3}$. (Students may either use a ruler to measure as precisely as possible, use paper folding, or estimate the distances. In the latter case, they will need to make sure their thirds are close to the same size.) **Approach 2:** Students may interpret $\frac{4}{3}$ times A as "4 times A divided by 3". In this case, they would quadruple the distance between 0 and A, then divide the distance between 0 and 4 times A into three equal parts and choose the point representing the first part. The picture below shows the point C as representing $\frac{4}{3}$ times A using Approach 1. Approach 2 will give you the same point, but students will need to extend the number line in order to place 4 times A on the number line. 	where to place $\frac{4}{3}$ x 6 on the number line? How can we use this idea to determine where to place Chelsea's point?
Part (c): To multiply the number A by $\frac{13}{6}$, students may approach the problem in a number of ways. **Approach 1:** Students may interpret $\frac{13}{6}$ times A as "$\frac{13}{6}$ of A". They would then take the distance between 0 and A on the number line, divide it into six equal spaces, then choose a point that represents $\frac{13}{6}$ of the distance between 0 and A. Note that students will need to extend	**Assessing Questions:** How did you determine where to place Darnell's point? How did you know that Darnell's point would be to the right of A on the number line? **Advancing Questions:** Suppose we know that the value of A is 6. How could we figure out where to place $\frac{13}{6}$ x 6 on the number line? How can we use this idea to determine where to place Darnell's point?

their “sixths” past A to the right in order to choose $\frac{13}{6}$, and in fact, $\frac{13}{6}$ of A is more than twice the distance between 0 and A. (Students may either use a ruler to measure as precisely as possible, use paper folding, or estimate the distances. In the latter case, they will need to make sure their thirds are close to the same size.)

Approach 2: Students may interpret $\frac{13}{6}$ times A as “13 times A divided by 6”. In this case, they would multiply the distance between 0 and A by 13 (extended the number line to do so), then divide the distance between 0 and 13 times A into six equal parts and choose the point representing the first part.

The picture below shows the point D as representing $\frac{13}{6}$ times A using Approach 1. Approach 2 will give you the same point, but students will need to extend the number line in order to place 13 times A on the number line.



Part (d): To multiply the number A by $\frac{5}{8}$, students may approach the problem in a number of ways.

Approach 1: Students may interpret $\frac{5}{8}$ times A as “$\frac{5}{8}$ of A”. They would then take the distance between 0 and A on the number line, divide it into eight equal spaces, then choose a point that represents $\frac{5}{8}$ of the distance between 0 and A. (Students may either use a ruler to

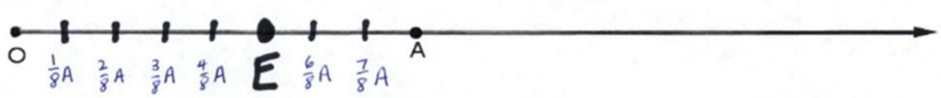
Assessing Questions:

How did you determine where to place Evan’s point?

How did you know?

Advancing Questions:

Suppose we know that the value of A is 8. How could we figure out where to place $\frac{5}{8}$ x 8 on the number line? How can we use this idea

measure as precisely as possible, use paper folding, or estimate the distances. In the latter case, they will need to make sure their eighths are close to the same size.) **Approach 2:** Students may interpret $\frac{5}{8}$ times A as "5 times A divided by 8". In this case, they would multiply the distance between 0 and A by 5, then divide the distance between 0 and 5 times A into eight equal parts and choose the point representing the first part. The picture below shows the point E as representing $\frac{5}{8}$ times A using Approach 1. Approach 2 will give you the same point, but students will need to extend the number line in order to place 5 times A on the number line. 	to determine where to place Evan's point? that Evan's point would be between 0 and A?
Part (e): Regardless of the size of A, the product of A and the other factor will fall between 0 and A on the number line whenever the second factor is between 0 and 1. One way to explain it is that if the second factor is between 0 and 1, then when we multiply A by that second factor, we are looking for that "part" of A (in the same way we were looking for $\frac{3}{5}$ of A in part (a)). Mathematically, we can "prove" this using inequalities. This approach appeals to techniques beyond fifth grade, but it is important to understand the mathematics behind the explanation: If F is our second factor and $0 < F < 1$, the multiplying all three parts of the inequality by the positive number A will give us:	**Assessing Questions:** Look at your answers to parts (a) – (d). When were your answers between 0 and A on the number line? When were your answers to the right of A on the number line? How did the factors help you decide where to place your points? **Advancing Questions:** How did you describe your factor to "force" the product of your factor and A to be between 0 and A on the number line? Does your description of the factor match your answers to parts (a)-(d)? Why or why not?

$0 * A < F * A < 1 * A$ or $0 < F * A < A$ so that F*A would be between 0 and A on the number line. (Note that if A is negative—a 6th grade and beyond concept—our inequalities would reverse and F * A would still be between 0 and A—except on the other side of the number line.)	
Part (f): Regardless of the size of A, the product of A and the other factor will fall to the right of A on the number line whenever the second factor is larger than 1. One way to explain it is that if the second factor is larger than 1, then when we multiply A by that second factor, we are looking for "more than" A. To more clearly understand this, we can use the fact that when the second factor is larger than 1, we can rewrite the second factor a 1 + "something". Multiplying this by A, we can use the distributive property of multiplication over addition to show that: A * our second factor = A * (1 + "something") = A * 1 + A * "something" = A + A * "something" which is larger than A. Mathematically, we can "prove" this using inequalities. This approach appeals to techniques beyond fifth grade, but it is important to understand the mathematics behind the explanation: If F is our second factor and F > 1, the multiplying both parts of the inequality by the positive number A will give us: $F * A > 1 * A$ or $F * A > A$ so that F*A would be to the right of A on the number line. (Note that if A is negative—a 6th grade and beyond concept—our inequalities would	**Assessing Questions:** Look at your answers to parts (a) – (d). When were your answers between 0 and A on the number line? When were your answers to the right of A on the number line? How did the factors help you decide where to place your points? **Advancing Questions:** How did you describe your factor to "force" the product of your factor and A to be between 0 and A on the number line? Does your description of the factor match your answers to parts (a)-(d)? Why or why not?

reverse and F * A would be less than A. On the number line, this would place F * A to the left of A, so that it would still be on the opposite side of A from 0.)	
Possible Student Misconceptions	
Students may have difficulty working with the point A on the number line. The point A represents an arbitrary point, and there are no other points (other than the origin 0) on the number line to show the relative size of A. The idea of A being an arbitrary point is a very abstract concept, and students may need something more concrete to begin the problem.	What would you do in part (a) if your number A represented 5 on the number line? How would you solve the problem? How would you solve the problem if your number A represented 10? How would this process change if your number A represented 12?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	What would you do in part (a) if your number A represented 5 on the number line? How would you solve the problem? How would you solve the problem if your number A represented 10? How would this process change if your number A represented 12?
If students finish early….	Are there other ways you could find your answers to parts (a)-(d)? What other ways could be used? Do these different strategies give you the same point on the number line every time?
Discuss/Analyze	
Whole Group Questions	
Key understanding: Multiplying a positive number A by a number between 0 and 1 results in product that is smaller than A. Multiplying a positive number A by a number larger than 1 results in a product that is larger than A.	
Questions:	
• *In parts (a)-(d), when was your product between 0 and A? When was your product larger than A? Is this always true? Why or why not?* • *Suppose we stretched a rubber band between 0 and A, then allowed the end that touched A to either stretch or shrink to our points representing the product of another factor and A. How would we know when the rubber band would shrink? How would we know when the rubber band would stretch?*	

Scaling Points

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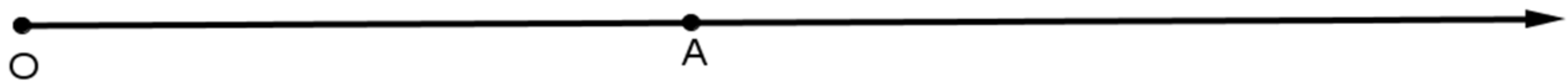

a) Bennie must multiply the number A by $\frac{3}{5}$. Where would Bennie's number be placed on the number line? Why?

b) Chelsea must multiply the number A by $\frac{4}{3}$. Where would Chelsea's number be placed on the number line? Why?

c) Darnell must multiply the number A by $\frac{13}{6}$. Where would Darnell's number be placed on the number line? Why?

d) Evan must multiply the number A by $\frac{5}{8}$. Where would Evan's number be placed on the number line? Why?

e) If the number represented by A is multiplied by another factor, when will the product of A and the other factor be between 0 and A on the number line? Why?

f) If the number represented by A is multiplied by another factor, when will the product of A and the other factor be to the right of A on the number line? Why?